EARTHQUAKE ENGINEERING AND STRUCTURAL DYNAMICS, VOL. 11, 135–146 (1983)

PERIODIC RESPONSE OF A SLIDING OSCILLATOR SYSTEM TO HARMONIC EXCITATION

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SUMMARY

This paper deals with the periodic response of an oscillating system which is supported on a frictional interface. The base excitation is assumed harmonic and the frictional force is assumed to be of the Coulomb type. Though each segment of the motion of such a system is described by linear equations, its complete response is highly non-linear and varied. The most fundamental periodic solutions are derived analytically and numerically. The results indicate that such a system has several subharmonic resonant frequencies and that while the friction reduces the peak response of the system when it is excited at its 'fixed-base' natural frequency, ω_n, the sliding can induce considerably higher levels of response, when compared with those of a non-sliding, fixed-base system, for frequencies less than ω_n. The results obtained herein may find application in the area of vibration isolation.

INTRODUCTION

This paper has been primarily motivated by the considerable interest in the earthquake engineering community to reduce the levels of response of structural and mechanical systems subjected to strong earthquake ground shaking. One of the vibration isolation methods that has received considerable interest in the recent past deals with the concept of mounting such systems on surfaces which permit sliding.[1] The rationale behind this is that if sliding could occur between the base of the system and the ground (or rubber pads, say, connected to the ground), the consequent slippage of the structure could reduce the peak displacements (relative to the base) of the system (Figure 1). Thus the stresses in the structure could be reduced at the expense of relative displacements between the structure and the ground. Though the plausibility of using such an isolation technique has often been proferred, very few, if any, analytical studies to date have been carried out to assess the merit of this idea for actual structural systems. Crandall *et al.*[2] have studied the problem of a rigid block supported on a sliding frictional interface subjected to random excitation and have shown that this philosophy may hold some promise. Caughey and Dienes[3] studied the response of a similar system to white noise excitation. Den Hartog[4] and Hundal[5] have used Coulomb friction as a damping mechanism to study the response of a single degree of freedom oscillator subjected to dynamic loads.

In this paper we extend the work of Crandall *et al.* and Caughey and Dienes, and study the response of a simple structural system (Figure 1) which rests on a frictional interface. The system chosen is generic in nature and it is anticipated that the analysis of this basic mechanics problem will yield a suitable framework

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0098–8847/83/010135–12$01.20

Received 10 February 1982
Revised 7 June 1982

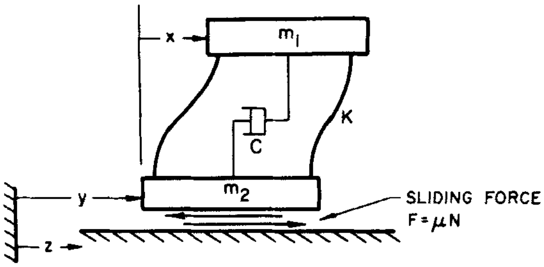


Figure 1. Coordinate systems for the sliding oscillator

for further studies of more complex systems. We mention here in passing that the dynamics of such a system would find application in several other areas of engineering practice such as in the design of mechanical equipment, especially, for large control valves in the nuclear industry.

The frictional resistance between the base and the structure is assumed to be of the Coulomb type and the coefficients of static and dynamic friction are taken to be the same. The frictional interface influences the excitation transferred to the system. It is found that the sliding mechanism could significantly alter the response characteristics of the oscillating structural system as compared to those of a rigid block. The non-linear system is shown to exhibit subharmonic resonant responses, the frequencies of which are related to those of the 'fixed-base', non-sliding oscillator.

THE MATHEMATICAL METHOD

The system is modelled by two masses m_1 and m_2 which are connected by a spring of stiffness k, and a dashpot with damping c (Figure 1). There is a Coulomb frictional force between the base (mass m_2) and the rigid horizontal plane such that sliding will occur when the shear force at the contact of the base mass with the supporting plane exceeds μN, where N is the total normal force ($N = g(m_1 + m_2)$). During the time intervals when sliding is not occurring, the system behaves as an oscillator excited by the motion of the rigid plane. During a sliding phase the mass m_2 experiences a Coulomb force, μN, of constant magnitude, independent of the plane's motion. Assuming that the static and dynamic friction coefficients are identical, the equations of motion for the two states can be written in terms of the relative displacement $x(t)$ as (Figure 1):

(1) during non-slip

$$\ddot{x} + 2\omega_n \xi \dot{x} + \omega_n^2 x = -\ddot{y}(t) = \ddot{z}(t) \tag{1}$$

with the condition

$$|\gamma\ddot{x} + \ddot{z}| \leqslant \mu g \tag{2}$$

where

$$\omega_n = \sqrt{(k/m_1)}, \quad \xi = \frac{c}{2\sqrt{(km_1)}}, \quad \gamma = \frac{m_1}{m_1 + m_2}$$

and,

(2) during slip

$$\ddot{x} + 2\omega_n' \xi' \dot{x} + \omega_n'^2 x = \frac{-\mu g}{1-\gamma} \operatorname{sgn}(\dot{z} - \dot{y}) \tag{3}$$

with

$$\ddot{y}(t) = \mu g \cdot \operatorname{sgn}(\dot{z} - \dot{y}) - \gamma\ddot{x} \tag{4}$$

where

$$\omega_n' = (1-\gamma)^{-\frac{1}{2}} \omega_n, \quad \text{and} \quad \xi_n' = (1-\gamma)^{-\frac{1}{2}} \xi$$

For large k, the spring will rigidly connect m_1 and m_2, and thus the system will tend to behave as a rigid block with a mass of m_1+m_2. For small k, the force on m_2 due to the relative motion $x(t)$ will be much less than the inertial forces on m_2 and the base will respond as a rigid block with mass equal to m_2 and an equivalent coefficient of friction $\mu' = \mu(m_1+m_2)/m_2$. Equations (3) and (4) when contrasted with equations (1) and (2) indicate that the system would exhibit different properties in the non-slip mode from those in the slip mode. Equation (4) indicates that the effective damping and stiffness in the slip condition have increased from the values k and c to $k(1-\gamma)^{-1}$ and $c(1-\gamma)^{-1}$ respectively.

HARMONIC RESPONSE OF A SLIDING RIGID MASS

As an introduction to the sliding oscillator problem, consider a rigid block of mass m resting on a flat, horizontal surface with the coordinate system as shown in Figure 2. Denoting the coefficient of sliding

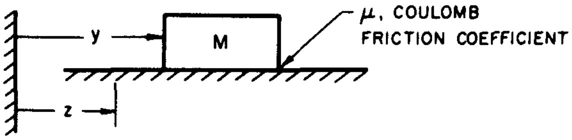


Figure 2. The sliding rigid block system

friction by μ and noting that the two forces acting on the block during sliding are the inertia force, $m\ddot{y}(t)$, and the sliding force, μgm, the condition for slip to occur is

$$|m\ddot{y}| = \mu gm \tag{5}$$

During the sliding mode

$$\ddot{y}(t) = \pm\mu g \tag{6}$$

and thus

$$\dot{y}(t) = \pm\mu g(t-t_0)+\dot{y}(t_0) \tag{7}$$

where t_0 is the time of initiation of the sliding. The sliding mode will cease at $t = t_f$ such that

$$\dot{z}(t_f) = \dot{y}(t_f) = \pm\mu g(t_f-t_0)+\dot{y}(t_0) \tag{8}$$

To demonstrate the various types of sliding response, the periodic solutions to a base acceleration

$$\ddot{z}(t) = a_0 \sin \omega t \tag{9}$$

will be determined with the condition for sliding to occur given by equation (5). It is seen that the condition for slippage to occur is independent of the frequency of the base motion and can be expressed as

$$\eta < 1, \quad \eta = \frac{\mu g}{a_0} \tag{10}$$

Once the block is sliding it will either stick or start sliding in the opposite direction at the end of the current sliding mode. For η close to unity, the sliding will stop before the ground acceleration exceeds μg for the next half cycle, and thus the periodic solutions will consist of two sliding and two non-sliding modes per cycle of input, as shown in Figure 3. These types of solutions will be referred to as slip–stick solutions. As η decreases, the duration of the sliding mode will increase until the time of the end of slip, t_1, exceeds the time of initiations

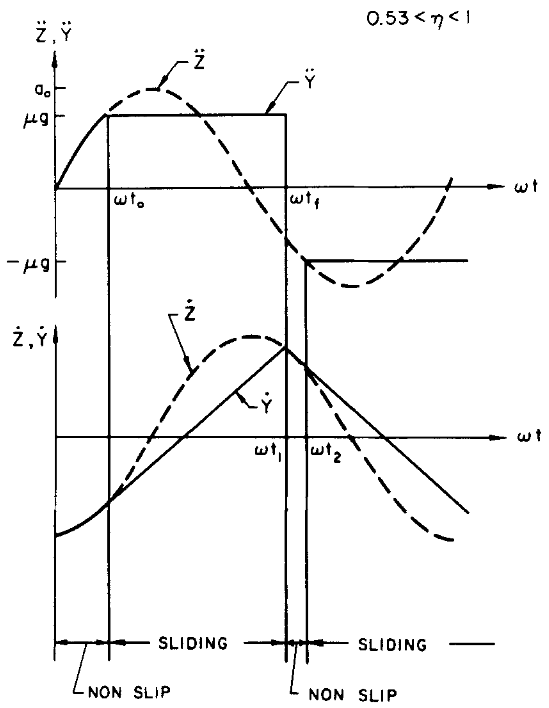


Figure 3. The accelerations and velocities for slip–stick motion of the rigid block.

of sliding in the opposite direction, t_2. For this case the response would consist of two sliding modes (in opposite directions) per cycle of base motion, as shown in Figure 4, and will be referred to as a slip–slip solution.

Consider one cycle of the base acceleration for the slip–stick mode of solutions, as shown in Figure 3. The condition for t_0, the initiation time of the sliding mode, is found from equations (6) and (9) and is

$$t_0 = \frac{1}{\omega}[\sin^{-1}(\pm\eta)\pm 2n\pi], \quad n = 0, 1, 2, \ldots \tag{11}$$

The condition for t_f, the time at the end of the sliding mode, is given by equation (8) along with equation (9), which yields

$$\cos\omega t_f \mp \eta\omega(t_f - t_0) = \cos\omega t_0 \tag{12}$$

As previously stated, for small η, t_f exceeds the initiation time for the next slip mode and so the periodic solutions will taken the form of those in Figure 4. The value of η for this transition is found by requiring that the times at which any two consecutive sticks and slips occur satisfy

$$\omega(t_f - t_0) = \pi \tag{13}$$

The numerical solution of equation (11), (12) and (13) gives

$$\eta = 0{\cdot}53 \tag{14}$$

By definition of the sliding mechanism the base acceleration, $\ddot{y}(t)$, is always bounded by

$$|\ddot{y}(t)| \leqslant \mu g \tag{15}$$

and $\dot{y}$ is bounded by

$$|\dot{y}(t)| \leqslant |\dot{z}(t)|_{\max} = \frac{a_0}{\omega} \tag{16}$$

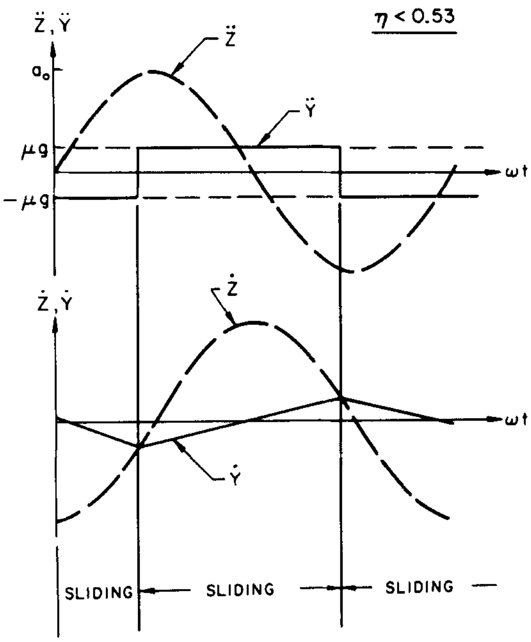


Figure 4. The accelerations and velocities for slip–slip motion of the rigid block

For the pure sliding solutions, $\eta < 0{\cdot}53$, the velocity of the block is bounded by its value at t_f (or t_0) as shown in Figure 4, and thus

$$|\dot{y}| \leqslant \frac{a_0}{\omega} |\cos \omega t_f| \tag{17}$$

where t_f is calculated from equations (12) and (13).

THE SLIDING OSCILLATOR RESPONSE

The replacement of the rigid block by the system of Figure 1, will alter the system state at which slip occurs since the base of the system will now experience the time dependent spring and dashpot forces of the oscillator. These forces can either inhibit or promote slippage of the base depending on the frequency of motion and the value of η. As seen from equations (1)–(4), the system has a different natural frequency and damping for its sliding and non-sliding modes. Since the condition for the continuation of a sliding mode is that $\dot{z} - \dot{y}$ does not change sign, it is clear that the response of m_1 relative to m_2 during a sliding mode is independent of the ground motion, z. However, it does depend on the initial conditions of $x(t)$ and $\dot{x}(t)$ (and therefore z) at the time when the sliding initiates. Thus, during a period of sliding the right hand side of equation (3) is a constant. Therefore, the response during the sliding modes is equivalent to the free vibration of a damped single degree of freedom oscillator vibrating about a displaced origin, and can be described by,

$$\ddot{\phi} + 2\xi' \omega_n' \dot{\phi} + \omega_n'^2 \phi = 0 \tag{18}$$

with

$$x(t) = \phi(t) - \frac{\mu g}{\omega_n'^2 (1-\gamma)} \cdot \operatorname{sgn}(\dot{z} - \dot{y}) \tag{19}$$

a. Determination of slip condition

Consider the input base acceleration, $\ddot{z}(t)$, to be harmonic and given by equation (9). For the rigid block it was seen that the block slipped only for $\eta < 1$; now, however, the sliding condition is also dependent on the oscillator parameters. To derive the maximum value of η for sliding to exist, the steady state solution for the non-slip condition [equation (1)] with $\ddot{z}$ given by equation (9), is derived as

$$\ddot{x}_{\text{steady state}} = a_0(\omega/\omega_n)^2[(1-(\omega/\omega_n)^2)^2+(2\xi\omega/\omega_n)^2]^{-\frac{1}{2}}\sin(\omega t-\theta) \tag{20}$$

with the phase, θ given by

$$\tan\theta = 2\xi\omega/\omega_n(1-(\omega/\omega_n)^2)^{-1} \tag{21}$$

Since sliding will occur when

$$|\gamma\ddot{x}+\ddot{z}| = \mu g \tag{22}$$

for slip to occur, η needs to be less than or equal to η^* where

$$\eta^* = (\beta^2+2\beta\cos\theta+1)^{\frac{1}{2}} \tag{23}$$

with

$$\beta = \gamma(\omega/\omega_n)^2\,[(1-(\omega/\omega_n)^2)^2+(2\xi\omega/\omega_n)^2]^{-\frac{1}{2}} \tag{24}$$

The values of η^* have been calculated ($\xi = 0{\cdot}02$) and are shown plotted vs. ω/ω_n for several values of the mass ratio, γ, in Figure 5. As the top mass, m_1, approaches zero the solution approximates that of a rigid block, $\eta = 1$, except near the resonant frequency of the system, where the top mass displacement becomes large and hence the force exerted on mass m_2 by the oscillation of mass m_1 becomes appreciable. The decrease in η^* for frequencies slightly larger than ω_n is due to the reversal in phase of the top mass relative to the base acceleration which decreases the total force on mass m_2. Thus, for $\omega/\omega_n > 1$, sliding of the base mass, m_2, may be inhibited by virtue of the oscillations of mass m_1. The phase agreement when $\omega < \omega_n$ increases the base force and thus the value of η^* for sliding.

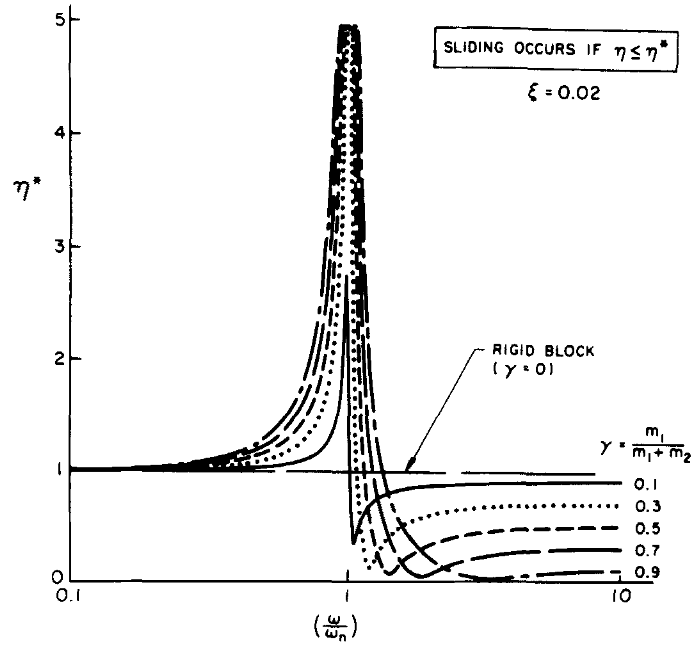


Figure 5. The maximum value of η, η^*, for steady state sliding of the oscillator versus the frequency ratio, ω/ω_n, for various mass ratios, γ, and $\xi = 0{\cdot}02$

As ω/ω_n approaches zero the oscillator becomes relatively stiff and the relative displacement of mass m_1 with respect to mass m_2 will become small for a fixed amplitude of excitation. In this case the system response will approach that of a rigid block, $\eta = 1$, as previously discussed. For $\omega/\omega_n \gg 1$, equation (1) becomes

$$\ddot{x}(t) = -\ddot{y}(t) \tag{25}$$

and the slip condition, equation (4), becomes

$$\ddot{y}(t) = \frac{\mu g}{(1-\gamma)} \cdot \operatorname{sgn}(\dot{z}-\dot{y}) \tag{26}$$

This is equivalent to the slip condition for a rigid block [equation (5)] with an effective friction coefficient, μ', of $\mu' = (1-\gamma)^{-1}$. Since rigid block sliding occurs at $\eta' = 1$, for $\omega/\omega_n \gg 1$ the values of η^* approach $(1-\gamma)$ as shown in Figure 5.

This frequency dependent behaviour of the parameter η^* can be viewed, for convenience, in terms of the transfer function of a 'slip-filter', so that for a given value of the dimensionless acceleration amplitude η, one can find the range of frequencies over which slippage could occur. It is seen that for $\eta \geq (1-\gamma)$, slippage occurs for $0 < a \leq \omega/\omega_n < b \leq \infty$ where the bandwidth, $(b-a)$, is always finite. For all frequencies outside this bandwidth, no slippage occurs. On the other hand, for $\eta < 1-\gamma$, excitation at all frequencies except for those lying in a finite bandwidth will lead to slippage.

b. The periodic solutions

Consider the periodic solutions to equations (1)–(4) with the period equal to that of $\ddot{z}(t)$, namely, $2\pi/\omega$. The general structures of perhaps the simplest of such possible solutions will be the two types noted for the sliding block problem; a slip–slip, or stick–slip response. It will also be assumed that the response consists of at most two slip intervals, in opposite directions, per cycle of input excitation. For large values of m_1/m_2 and for $\omega/\omega_n \ll 1$ there can exist numerous slip–non-slip intervals per half cycle.

To derive the equation for the slip–slip periodic solutions we start with a solution in terms of a general initial time t_0 and piece together two segments of slip motion to obtain a periodic slip–slip motion. Using equation (18) and (19), the response, $x(t)$, during either slip interval is written in terms of the initial conditions at the beginning of that slip interval, $\mathbf{x}(t_0)$, as

$$\mathbf{x}(t) \triangleq \begin{Bmatrix} x(t) \\ \dot{x}(t) \end{Bmatrix} = \mathbf{A}(t-t_0)\,x(t_0) + \mathbf{b}(t-t_0) \tag{27}$$

where

$$\mathbf{A}(\tau) = e^{-\xi'\omega_n'\tau}\mathrm{S}\begin{bmatrix} \xi(1-\xi^2)^{-\frac{1}{2}}\sin\omega_d'\tau + \cos\omega_d'\tau & \sin(\omega_d'\tau)/\omega_d' \\ -\omega_n'(1-\xi^2)^{-\frac{1}{2}}\sin\omega_d'\tau & \cos\omega_d'\tau - \xi(1-\xi^2)^{-\frac{1}{2}}\sin\omega_d'\tau \end{bmatrix}$$

and

$$\mathbf{b}(\tau) = \begin{Bmatrix} \alpha\,\operatorname{sgn}(\dot{z}-\dot{y})\,[(\xi(1-\xi^2)^{-\frac{1}{2}}\sin\omega_d'\tau + \cos\omega_d'\tau)\,e^{-\xi'\omega_n'\tau} - 1] \\ -\alpha\,\operatorname{sgn}(\dot{z}-\dot{y})\,\omega_n'(1-\xi^2)^{-\frac{1}{2}}\,e^{-\xi'\omega_n'\tau}\sin\omega_d'\tau \end{Bmatrix}$$

with

$$\alpha = \frac{-\mu g}{\omega_n'^2(1-\gamma)} \tag{28}$$

and,

$$\omega_d' = (1-\xi^2)^{\frac{1}{2}}\,\omega_n'$$

Assuming that the slippage continues until time t_f, the slip–slip periodic solutions then satisfy

$$\omega(t_f - t_0) = \pi \tag{29}$$

Further, the periodicity of the solution implies that

$$\mathbf{x}(t_f) = -\mathbf{x}(t_0) = \mathbf{A}(\pi/\omega)\ \mathbf{x}(t_0) + \mathbf{b}(\pi/\omega) \tag{30}$$

which yields,

$$\mathbf{x}(t_0) = -[\mathbf{A}(\pi/\omega) + \mathbf{I}]^{-1}\,\mathbf{b}(\pi/\omega) \tag{31}$$

The solutions for $x(t_0)$ calculated from the above equation and scaled by α are shown plotted vs. ω/ω_n in Figure 6 for various ξ values. It is seen from this figure that the system exhibits a set of response peaks for

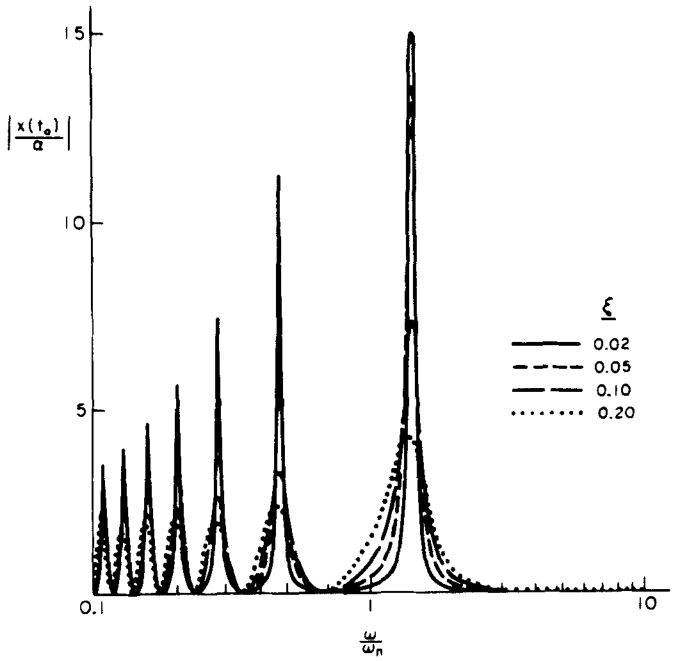


Figure 6. The top displacement at the onset of each slip mode for the periodic slip–slip solutions

$\omega/\omega_n < 1$. These resonant frequencies can be simply derived for zero damping ($\xi = 0$) from the zeroes of the determinant of $[\mathbf{A}(\pi/\omega) + \mathbf{I}]$. From equation (28) it is found that

$$\|\ \mathbf{A}(\pi/\omega) + \mathbf{I}\ \| = [\cos(\pi\omega_d'/\omega) + 1]^2 \tag{32}$$

the zeroes of which occur at $\omega = \omega_d'/(2n+1)$, $n = 0, 1, 2, \ldots$.

The growth of the solutions at these subharmonic frequencies is expressible from equation (30) where

$$x(t_f) = \cos\{\omega_d'(t_f - t_0)\}\, x(t_0) + \alpha\, \mathrm{sgn}\,(\dot{z} - \dot{y})\,[\cos\{\omega_d'(t_f - t_0)\} - 1] \tag{33}$$

At $\eta = 1, 3, 5, \ldots$, the value of $\omega(t_f - t_0)$ approaches π, and thus equation (33) becomes

$$x(t_f) \simeq -x(t_0) - 2\alpha\, \mathrm{sgn}\,(\dot{z} - \dot{y}) \tag{34}$$

for $x(t_0)$ large. Over a full period then the initial displacements for these undamped solutions will grow to

$$|x(t_0+2\pi/\omega)| = |x(t_0)|+4\alpha \tag{35}$$

Therefore, similar to the harmonic resonance of an undamped linear oscillator, the peak response per cycle, or the amplitude envelope, grows linearly in time. As seen from the form of the solution in equation (31) $x(t)$ is linearly dependent on μ and inversely dependent on the mass ratio $(1-\gamma)$. As the acceleration of the top mass, $\ddot{x}(t)$, grows during this resonance, $\ddot{y}(t)$ also grows, but the two are out of phase in order to satisfy equation (4). The slip–slip type of periodic solution can exist only for η less than some particular value, say η^*, as with the rigid block solutions, and this η^* must be less than or equal to the value of η for any form of sliding solution as given in Figure 5. Since the system will slip for larger values of η at $\omega/\omega_n > 1$ (Figure 5), and the peak response is generally much greater for $\omega/\omega_n < 1$, the system is qualitatively more sensitive to frequencies lower than the natural frequency, ω_n, of the fixed-base system. Examples of these periodic solutions are shown in Figure 7 for three frequency ratios.

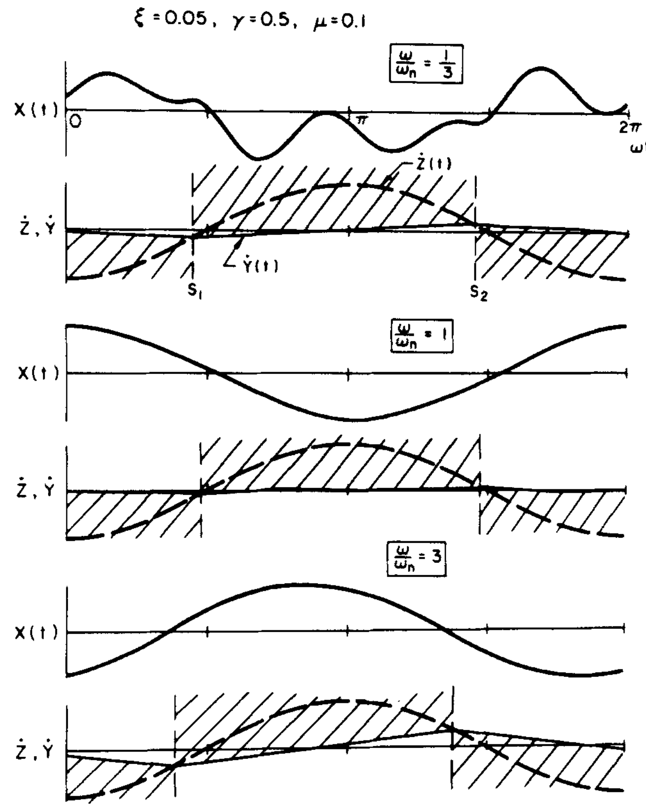
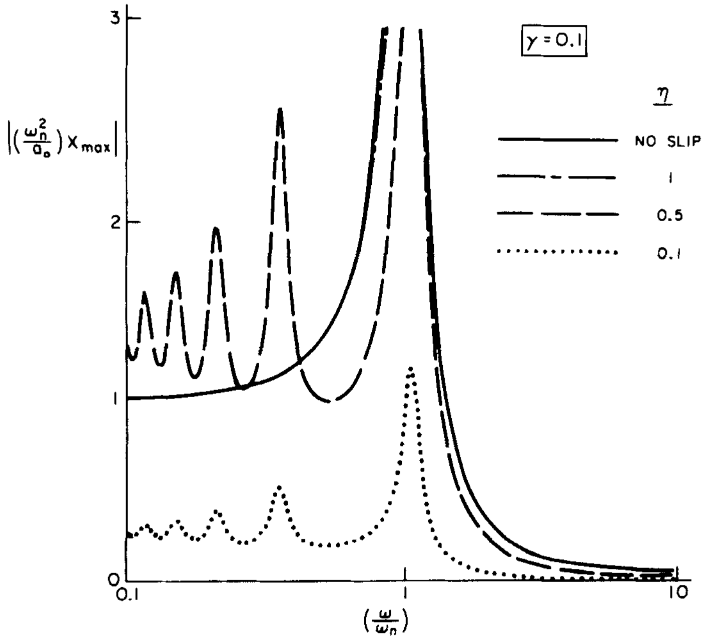


Figure 7. The top displacement, $x(t)$, and the ground and base velocities, $\dot{z}(t)$ and $\dot{y}(t)$, of periodic solutions for $\xi = 0{\cdot}05$, $\mu = 0{\cdot}1$ and $\gamma = 0{\cdot}5$. The shaded regions denote the domains of $\dot{z}(t)$ for the $x(t)$ response shown

We note that one can visualize several aperiodic and periodic accelerations, $\ddot{z}(t)$, all of which would yield identical responses, $x(t)$. As an example, all of the analytic functions $\dot{z}(t)$ which lie in the shaded regions shown in Figure 7 and which have identical values of $\ddot{z}(t)$ and $\dot{z}(t)$ at the slip reversal points s_1 and s_2 would yield the same response, $x(t)$. The input, then, is not uniquely determinable from the response, making the inverse problem virtually unsolvable. This suggests that evaluating the peak acceleration amplitudes of strong earthquake ground motion from the observed response of sliding objects would be difficult, particularly for objects that were caused to slip for a majority of the time. Unlike linear and some non-linear dynamic models the response is not as dependent on the peak amplitude of $\ddot{z}(t)$ as on some other norm of $\ddot{z}(t)$ which represents the relative amount of sliding and the conditions of the system at the onset of slip.

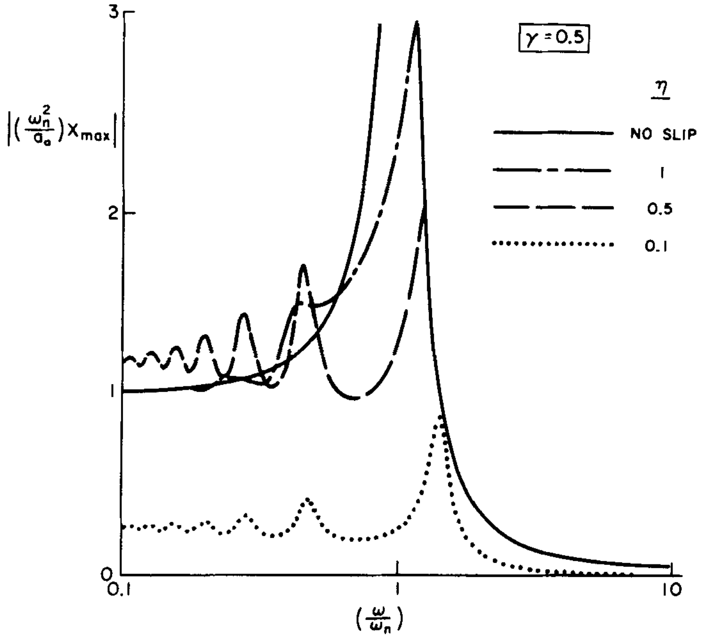
In a similar manner to the slip–slip type, other periodic solutions can be derived by proposing a sliding–non-sliding behaviour and constructing the solution transformations for each mode involving the continuity and periodicity conditions. Unlike the slip–slip solutions, however, the resulting equations will be transcendental and must be solved numerically. To determine the totality of periodic solutions, it is best to take advantage of the linear structure of the solution within each sliding or non-sliding interval. For such linear equations with damping present it can be shown that the solutions are asymptotically stable about the steady solution since the displacements and velocities are continuous at all times.[6] The periodic solutions for the damped systems can thus be determined by numerically calculating the transient solution to equations (1) through (4) with arbitrary initial conditions, and the solution will approach the periodic solution as time increases. This method was employed to calculate numerically the peak responses of the solutions versus ω/ω_n for $\xi = 0{\cdot}05$ [Figures 8(a), 8(b) and 8(c)].

For low values of η the solutions are of the slip–slip type and exhibit the multiple peaks at frequencies lower than ω_n. At $\omega/\omega_n \ll 1$ the forces exerted on mass m_2 due to oscillations of mass m_1 are much larger than the inertial force for non-sliding, $m_2 \ddot{z}$, and the sliding is primarily perpetuated by the oscillations of mass m_1. Basically then the transient part of the motion would consist of the build up of the energy in the oscillator by the input motion, and at the steady state, the input energy will balance the slip and viscously dissipated energy. The energy of the oscillator, however, may be greater than that in one which is not sliding. For $\omega/\omega_n \gg 1$ the inertial force on m_2 will be greater than the force on m_2 due to the oscillations of mass m_1, and thus m_2 will behave as a rigid block. In the limit $x(t)$ will approach $-y(t)$, the negative of the displacement of m_2. From Figure 8 it is seen that the peak response of the non-sliding oscillator bounds the sliding response for this frequency range. For frequencies slightly larger than ω_n the sliding system response equals the non-sliding oscillator response for some frequency range. This range corresponds to the frequencies in Figure 5 over which a low value of η is necessary to maintain slip for the periodic solutions, again pointing out the greater sensitivity for the oscillator to slide at low values of ω/ω_n compared to the range $\omega/\omega_n > 1$.

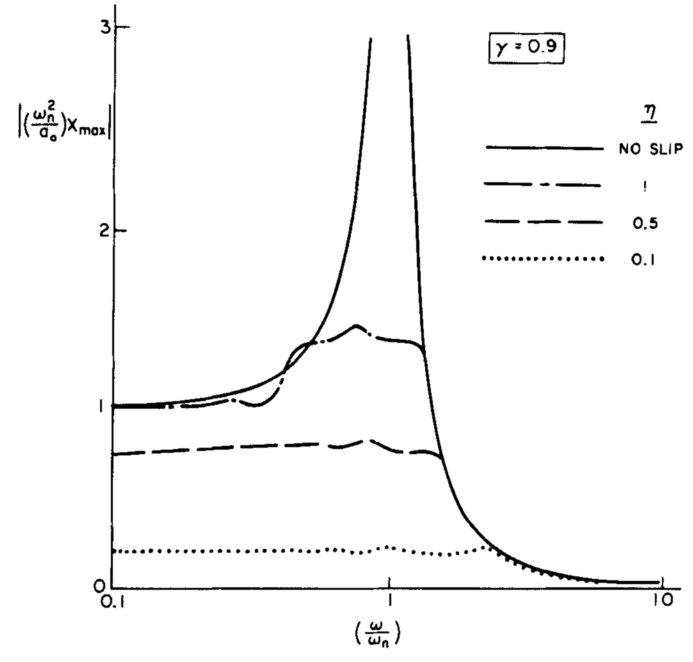


(a)

Figure 8. The peak displacement response of the periodic solutions versus ω/ω_n for $\gamma = 0{\cdot}1$ (a), $\gamma = 0{\cdot}5$ (b), and $\gamma = 0{\cdot}9$ (c), and $\xi = 0{\cdot}05$



(b)



(c)

Figure 8. (continued)

CONCLUSIONS AND DISCUSSION

The analysis of the periodic solutions of the harmonically excited system investigated herein, exhibits several interesting aspects.

(1) Whereas the condition for the initiation of slip for a rigid block resting on a frictional interface is that the base acceleration amplitude, a_0, equals μg, the condition for the initiation of slippage for the system under study here is frequency dependent. In fact it is shown that for values of a_0 considerably less than $\mu g(\eta > 1)$, slippage can still prevail provided the excitation lies in a suitable frequency band. The effect of the oscillatory system on the initiation of slip is manifested by the curves of Figure 5. Sliding initiates at higher values of η (and therefore lower values of a_0) for the frequency range $\omega/\omega_n < 1$, and for lower values of η for the range $\omega/\omega_n > 1$. When $\omega/\omega_n \gg 1$, the system behaves similar to a rigid block and the response, $x(t)$, is bounded by the non-sliding response. The fundamental resonant peak of the system near $\omega/\omega_n \simeq 1$ occurs at larger values of ω/ω_n as η decreases (Figure 8).

(2) The system shows several subharmonic resonant frequencies so that the peak response, $x(t)$, of such a system to harmonic excitation may not necessarily be less than that of a non-sliding, fixed-base system.

(3) The nature of the system response to base excitations shows that, in fact, various periodic and aperiodic inputs could yield identical responses. Thus the inverse problem of the determination of the peak ground acceleration from records of the system response, $x(t)$, may lead to non-unique solutions.

(4) Although only the periodic solutions have been discussed, numerical calculations for a variety of frequencies, damping ratios and mass ratios showed that the solutions converged quite rapidly to the periodic solutions for most cases (usually within 15 cycles for zero initial conditions), indicating that the periodic solutions are a significant part of the response to harmonic excitations. However, for a dynamic system such as this it is difficult to infer any information about the transient response to non-periodic excitation (e.g., earthquake motion) from the period response. The multiple response peaks at $\omega/\omega_n < 1$ may not contribute significantly to the transient response since their existence depends heavily on the periodicity of the excitation. The transient response of such a system will be discussed in a paper to follow.

ACKNOWLEDGEMENT

We wish to express our appreciation to Professor Paul Seide and Dr. Marijan Dravinski for their comments and suggestions. We thank Professor T. Caughey of the California Institute of Technology for his comments on the manuscript. This work was funded in part by funds from the National Science Foundation.

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