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Trade-Offs Between Identification and Control in Dynamic Systems

A quantitative study of the trade-offs between the tasks of control law design and plant identification for linear dynamic systems is presented. The problem is formulated in the context of optimal control and optimal identification through the intermediary concept of an optimal input. The duality between identification and control is quantified by optimal inputs, which have a specified amount of energy, and which minimizes the objective function. The optimization problem together with the energy constraint is formulated by using an augmented state vector. This results in a nonlinear two-point boundary value problem and eliminates the need for using a trial and error approach to satisfy the energy constraint. An example of a single-degree-of-freedom oscillator is used to illustrate the basic concepts underlying the proposed approach. Significant trade-offs between identification and control tasks are observed, the trade-offs becoming increasingly important for increasing levels of input energy.

Introduction

The control of large flexible structures is an area that has attracted considerable interest in recent years from both the professional and the research community (Meirovitch (1985); Rodriguez (1985); Aubrun (1980); Benhabib et al. (1981); Meirovitch (1982)). This interest is primarily motivated by the need to precisely control flexible structures in various developing fields of modern technology. In the area of earthquake engineering, the reduction of structural response may be necessary to reduce internal stresses caused by dynamic loads thereby reducing the damage potential and increasing the useful life of structures (Martin and Soong (1976)). In space applications, the availability of the Space Shuttle to transport large payloads into orbit at reasonable costs presents an opportunity for large systems to perform new missions in space. However, because of launch weight and volume constraints, these structures are generally very flexible and pose new challenges in all aspects of control such as attitude control and maneuvering, precision pointing, vibration attenuation, and structural and shape control (AIAA (1978)).

A necessary prelude to the effective control of a structure is a knowledge of its characteristics and properties. In other words, one needs to have information about the structural system so that adequate control algorithms can be devised. This has led to a considerable interest in the identification of structures subjected to dynamic loads (Udwadia (1985); Graupe (1972); Mehra and Lainiotis (1976); Tung (1981); Dale and Cohen (1971)). For structural systems that are described by parametric models, this involves knowledge of the nature of the governing differential equations and knowledge of the values of the parameters that are involved or, at least, knowledge of the bounds within which the parameters lie. Clearly, the better the system is identified (the smaller the bounding intervals within which the parameters are known to lie), the more finely tuned the controller can be made, so that for a given amount of available control energy, the control would be more efficient. The less knowledge we have about the structural system the more robust the controller needs to be and, in general, the less efficient the control for a given amount of available control energy. Thus, heuristically speaking, there exists a duality between the concepts of identification and control, because (1) robust controllers may require reduced efforts at identification (for purposes of control), and (2) increased efforts at identification may require less robust and more efficient controllers. However, the tradeoffs between identification and control, from a practical standpoint, are still usually difficult to assess quantitatively. Little work has been reported to date in this area of quantitative cost-benefit analysis between these two dual concepts. A simulation study comparing adaptive control and identification algorithms is given in Benhabib and Tung (1980), though, here again, no quantitative results were presented.

In this paper we formulate the trade-off problem between identification and control, and study in a *quantitative* manner their duality through the use of the intermediary concept of an optimal input. Thus, the paper attempts to answer the following question: Given that the optimal input time function is to have a certain prescribed energy, how does it change in character and in its effectiveness as one changes the objective criterion from one that emphasizes control to one that emphasizes identification? While the analytical work presented here has been motivated by our need to control flexible structural systems the methodology developed and the results obtained are applicable to all systems governed by ordinary dif-

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ferential equations; thus it applies to the general multivariable and multiparameter systems commonly encountered in chemical, civil, electrical, and mechanical engineering. Some simple numerical examples are provided to indicate the quantitative nature of the results and provide a feel for them. We believe that these examples capture the important aspects of the problem without unnecessary complication of the concept. The results in these examples show that significant trade-offs exist between identification and control and that for the same amount of energy in the input signal, the emphasis on control could lead to very high covariances of the parameter estimates. Similarly, inputs that are optimal for identification could yield responses whose mean squared values may be several times those obtained for inputs that yield optimal control.

Problem Formulation

Consider a dynamic system modelled by the first order set of differential equations

$$\dot{\mathbf{x}}(t) = \mathbf{F}_1 \mathbf{x} + \mathbf{G}_1 \mathbf{f} \qquad \mathbf{x}(0) = \mathbf{x}_0 \tag{1}$$

$$\mathbf{z}(t) = H_1\ \mathbf{x}(t) + v(t) \tag{2}$$

where $\mathbf{x}$ is an $x \times 1$ state vector, $\mathbf{f}$ is an $m \times 1$ control vector, $\mathbf{z}$ is an $r \times 1$ measurement vector and the $n \times 1$ initial condition vector, $\mathbf{x}_0$, is given. We shall assume that the measurement noise is representable as a zero mean Gaussian White Noise process so that

$$E[v(t)] = 0, \text{ and,} \tag{3}$$

$$E[v(t)v(\tau)] = R_1 \delta(t-\tau). \tag{4}$$

Let the vector of unknown parameters in the system modelled by equations (1) and (2) be given by the $p \times 1$ vector $\mathbf{\Theta}$. Let us assume that the identification is carried out with an efficient unbiased estimator so that the covariance of the estimate of $\mathbf{\Theta}$ namely $\hat{\mathbf{\Theta}}$ is provided by the inverse of the Fisher Information Matrix [13]. Hence,

$$\mathrm{Cov}[\hat{\mathbf{\Theta}}] = \mathbf{M}^{-1}. \tag{5}$$

The matrices $\mathbf{F}_1$ and $\mathbf{G}_1$ are taken to be functions of, in general, the parameter vector $\mathbf{\Theta}$. The optimal input for identification of the parameter vector $\mathbf{\Theta}$ is then sought such that a suitable norm related to the matrix $\mathbf{M}$ is maximized or minimized. Different measures of performance related to $\mathbf{M}$ have been used in the literature Mehra (1974):

(1) A-Optimality, where $\mathrm{Tr}(\mathbf{M}^{-1})$ is minimized;
(2) D-Optimality, where the determinant of $\mathbf{M}^{-1}$ is minimized; and,
(3) E-Optimality, where the maximum eigenvalue of $\mathbf{M}^{-1}$ is minimized.

In this paper, for expository purposes, we shall use the criterion for obtaining the optimal inputs for identification as the maximization of the Trace $\{\mathbf{W}^{1/2}\mathbf{M}\ \mathbf{W}^{1/2}\}$ where $\mathbf{W}$ is a suitable positive definite weighting matrix. Thus, the criterion for obtaining the optimal input for paramter identification is taken to be

$$J_I = \int_0^T \mathrm{Trace}\ \{\Psi_p^T H_1^T R_1^{-1} H_1 \Psi_p\}\ dt \tag{6}$$

where

$$\Psi_p = \mathbf{X}_p\ \mathbf{W}^{1/2}, \tag{7}$$

and the matrix $\mathbf{X}_p$ is given by,

$$[\mathbf{X}_p]_{i,j} = \frac{\partial \mathbf{x}_i}{\partial \mathbf{\Theta}_j} \tag{8}$$

In addition to the objective function generated by our need for identification, the objective function required to be maximized for control is,

$$J_C = -\int_0^T \{\mathbf{x}^T \mathbf{Q}_1 \mathbf{x}\}\ dt. \tag{9}$$

Here $\mathbf{Q}_1$ is a symmetric positive definite, $n \times n$ weighting matrix. This then yields the composite objective function which is required to be maximized as [13]

$$J = -(\alpha/2) \int_0^T \{\mathbf{x}^T \mathbf{Q}_1 \mathbf{x}\}\ dt + (\beta/2) \int_0^T \mathrm{Trace}\ \{\Psi_p^T \mathbf{H}_1^T \mathbf{R}_1^{-1} \mathbf{H}_1 \Psi_p\} dt \tag{10}$$

where α and β are positive scalars. Clearly, when $\alpha >> \beta$, finding $\mathbf{f}(t)$ to maximize J is tantamount to finding the optimal control for the system (1)-(2), while when $\beta >> \alpha$, the $\mathbf{f}(t)$ that maximizes J is simply the optimal input for identification of the $p \times 1$ parameter vector $\mathbf{\Theta}$. In particular, when $\alpha = 0$, and $\beta = 1$, the optimal input for "best" identification is obtained; when $\alpha = 1$, and $\beta = 0$, the optimal input for "best" control is obtained. Denoting the $n \times 1$ vector

$$\mathbf{x}_{\Theta_i} = \frac{\partial \mathbf{x}}{\partial \mathbf{\Theta}_i} \tag{11}$$

and assuming that the matrix $\mathbf{W}$ is diagonal, so that,

$$\mathbf{W} = \mathrm{Diag}(w_1, w_2, \ldots, w_p) \tag{12}$$

we can generate an augmented $n(p+1)$ vector,

$$y(t) = \begin{bmatrix} \mathbf{x}(t) \\ w_1^{1/2}\ \partial \mathbf{x}(t)/\partial/\mathbf{\Theta}_1 \\ w_2^{1/2}\ \partial \mathbf{x}(t)/\partial \mathbf{\Theta}_2 \\ \cdot\ \cdot \\ w_p^{1/2}\ \partial \mathbf{x}(t)/\mathbf{\Theta}_p \end{bmatrix}, \text{ with, } y(0) = \begin{bmatrix} x_0 \\ 0 \\ 0 \\ \cdot \\ 0 \end{bmatrix} \tag{13}$$

which is then governed by the differential equation

$$\dot{y} = \mathbf{F}y + \mathbf{G}\mathbf{f},\ y^T(0) = \{\mathbf{x}_0^T, 0\} \tag{14}$$

where $\mathbf{F}$ is the $n(p+1) \times n(p+1)$ matrix given by

$$\mathbf{F} = \begin{bmatrix} \mathbf{F}_1 & 0 & 0 & 0 & 0 \\ w_1^{1/2}\ \mathbf{F}_{\Theta_1} & \mathbf{F}_1 & 0 & 0 & 0 \\ w_2^{1/2}\ \mathbf{F}_{\Theta_2} & 0 & \mathbf{F}_1 & 0 & 0 \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ w_p^{1/2}\ \mathbf{F}_{\Theta_p} & 0 & 0 & 0 & \mathbf{F}_1 \end{bmatrix}, \tag{15}$$

and,

$$\mathbf{G} = \begin{bmatrix} \mathbf{G}_1 \\ w_1^{1/2}\ \mathbf{G}_{\Theta_1} \\ w_2^{1/2}\ \mathbf{G}_{\Theta_2} \\ \cdot \\ w_p^{1/2}\ \mathbf{G}_{\Theta_p} \end{bmatrix}, \tag{16}$$

with

$$\mathbf{F}_{\Theta_i} = \frac{\partial \mathbf{F}_1}{\partial \mathbf{\Theta}_i},\ \mathbf{G}_{\Theta_i} = \frac{\partial \mathbf{G}_1}{\partial \mathbf{\Theta}_i},\ i = 1, 2, \ldots, p. \tag{17}$$

We note in passing that the stability of the equation set (14) is controlled by the stability of the equation set (1) since the eigenvalues of the matrix $\mathbf{F}$ are identical to those of the matrix $\mathbf{F}_1$, as seen in equation (15), except for the increased multiplicities. The objective function (10) can now be rewritten, after some algebra, as

$$J = -(\alpha/2)\int_0^T y^T \mathbf{Q} y^T dt + (\beta/2)\int_0^T y^T \mathbf{H}^T \mathbf{R}^{-1} \mathbf{H} y dt \quad (18)$$

where, the matrices $\mathbf{Q}$, $\mathbf{H}$, and $\mathbf{R}^{-1}$ are the block diagonal matrices given by

$$\begin{aligned} &\mathbf{Q} = \text{Diag}\{\mathbf{Q}_1, 0, 0, \ldots, 0, 0\}, \\ &\mathbf{H} = \text{Diag}\{0, \mathbf{H}_1, \mathbf{H}_1, \mathbf{H}_1, \ldots, \mathbf{H}_1\}, \text{ and,} \\ &\mathbf{R}^{-1} = \text{Diag}\{\mathbf{R}_1^{-1}, \mathbf{R}_1^{-1}, \mathbf{R}_1^{-1}, \ldots, \mathbf{R}_1^{-1}\} \end{aligned} \quad (19)$$

Thus, the objective function needs to be maximized under the constraint equations (14) and the energy constraint

$$\int_0^T \mathbf{f}^T \mathbf{f} \, dt = E, \quad (20)$$

where the parameter E is given *a priori*.

The assumption of finite energy constraint, given in equation (20), for the control signal is a crucial step in the analysis that puts the trade-off study between identification and control in a meaningful context. From an analytical point of view, however, the constraint in equation (20) converts the standard linear two point boundary value problem to a nonlinear two point boundary value problem, thus making the numerical solution of the optimization problem much more involved.

Determination of Optimal Inputs for Simultaneous Control and Identification

Having formulated the problem for constrained maximization, we next use the standard Lagrange multiplier method to obtain the function $\mathbf{f}(t)$ which maximizes the objective (18) subject to (14) and (20). Using the Lagrange multipliers $\lambda(t)$ and $\eta(t)$ we therefore obtain the augmented objective function to be

$$\bar{J} = -\frac{\alpha}{2}\int_0^T \{y^T \mathbf{Q}_y\} \, dt + \frac{\beta}{2}\int_0^T \{y^T \mathbf{H}^T \mathbf{R}^{-1} \mathbf{H} y\} \, dt$$

$$+ \int_0^T \lambda^T(t) \, \{\mathbf{F}y + \mathbf{G}\mathbf{f} - \dot{y}\} \, dt + \int_0^T \frac{\eta(t)}{2} (\mathbf{f}^T \mathbf{f} - \dot{y}_{N+1}) \, dt \quad (21)$$

Here we augmented the state vector by the variable

$$y_{N+1}(t) = \int_0^t \mathbf{f}^T(\tau) \mathbf{f}(\tau) d\tau$$

where $N = n(p+1)$ and we use the additional Lagrange multiplier $\eta(t)$. By doing this we can satisfy the equality constraint (20) without having to resort to the usual trial and error procedure.

Taking the first variation, we obtain the following set of equations for the variables $y(t)$, $\lambda(t)$ and $\eta(t)$ (see Appendix):

$$\begin{aligned} &\dot{y}(t) - \mathbf{F}y = \frac{1}{\eta} V\lambda(t), \ y(0) = y_0 \\ &\dot{\lambda}(t) + \mathbf{F}^T \lambda(t) = -\alpha\{\mathbf{Q}\}y + \beta\{\mathbf{H}^T \mathbf{R}^{-1} \mathbf{H}\}y, \ \lambda(T) = 0, \\ &\dot{y}_{N+1} = \mathbf{f}^T \mathbf{f} = \frac{1}{\eta^2} \lambda^T(t) V \lambda(t), \ y_{N+1}(0) = 0, \ y_{N+1}(T) = E \\ &\dot{\eta}(t) = 0 \end{aligned} \quad (22a)$$

where $V = (\mathbf{G}\mathbf{G}^T)$. We note that the equation set (22*a*) constitutes a nonlinear two point boundary value problem containing $2[np + n + 1]$ first-order differential equations. The optimal input vector, $\mathbf{f}(t)$, is obtained through the solution of this two point boundary value problem using the relation:

$$\mathbf{f}(t) = -\frac{1}{\eta} \mathbf{G}^T \lambda(t) \quad (23a)$$

It is interesting to note that had we used the objective function

$$J_C{}' = \left[\int_0^T \mathbf{x}^T \mathbf{Q}_1 \mathbf{x} \, dt\right]^{-1} \quad (24)$$

instead of J_C in equation (9) we would obtain,

$$J' = \frac{\alpha}{2} J_C{}' + \frac{\beta'}{2} J_I \quad (25)$$

where J_I is given in equation (6), and the relative weighting of the contributions of the control and the identification objectives are denoted by α and β'. Following the same procedure as before and using the augmented vector $\mathbf{y}$, maximization of this objective function, J', along with the constraints (14) and (20) would then yield the following set of equations:

$$\begin{aligned} &\dot{\mathbf{y}}(t) - \mathbf{F}\mathbf{y}(t) = \frac{1}{\bar{\eta}} V\bar{\lambda}(t), \ \mathbf{y}(0) = \mathbf{y}_0, \\ &\dot{\bar{\lambda}}(t) + \mathbf{F}^T \bar{\lambda}(t) = -\alpha\{\mathbf{Q}\}\mathbf{y} + (\beta'\Delta)\{\mathbf{H}^T \mathbf{R}^{-1} \mathbf{H}\}\mathbf{y}, \ \bar{\lambda}(T) = 0, \\ &\dot{\mathbf{y}}_{N+1} = \frac{1}{\bar{\eta}^2} \bar{\lambda}^T(t) V \bar{\lambda}(t), \ \mathbf{y}_{N+1}(0) = 0, \ \mathbf{y}_{N+1}(T) = E \\ &\dot{\bar{\eta}}(t) = 0 \end{aligned} \quad (22b)$$

where, Δ is the positive quantity defined by

$$\Delta = \left[\int_0^T \mathbf{y}^T \mathbf{Q} \mathbf{y} \, dt\right]^2 = \left[\int_0^T \mathbf{x}^T \mathbf{Q}_1 \mathbf{x} \, dt\right]^2 \quad (26)$$

The optimal input is obtained from the relation

$$\mathbf{f}(t) = -\frac{1}{\bar{\eta}} \mathbf{G}^T \bar{\lambda}(t). \quad (23b)$$

Comparing equations (22*a*) and (22*b*), we observe that the only difference that arises in the use of the objective function (25) instead of (18) is the effective change in weighting parameter β. Setting $\beta'\Delta = \bar{\beta}$ the equation set (22*b*) becomes identical with the set (22*a*). The Lagrange multipliers $\lambda(t)$ of (22*a*) and λ of (22*b*) are related by $\bar{\lambda} = \lambda\Delta$; similarly, $\bar{\eta} = \eta\Delta$. It should be noted that the two equations become identical only when the function $\mathbf{y}(t)$ in (26) corresponds to the response for the optimal input $\mathbf{f}(t)$, i.e., the solution $\mathbf{y}$, of the set (22*a*). With this rescaling of the parameter β', the optimal input $\mathbf{f}(t)$ is identical for the two objective functions J and J' of (10) and (24). Having thus shown the quasi-equivalence of the two objective functions (10) and (25) through this rescaling, in this sequel we shall illustrate our results by using the objective function in the form of equation (10) which leads to the boundary value problem described in equation (22*a*).

This two-point boundary value problem can be numerically solved in various ways. An extensive literature on numerical techniques for solving such problems is available (Roberts and Shipman (1972)). Among the methods most commonly used are multiple shooting techniques [15] with Newton-Raphson iterations, and the Kalaba Method (Kagiwada and Kalaba (1968)) where the two point boundary value problem is converted to an equivalent Cauchy initial value problem. In this sequel, the equation set (22) is solved using the multiple shooting technique with the Newton iteration method.

Illustrative Examples

To exemplify the *concepts* developed, let us consider a system modelled by a single-degree-of-freedom oscillator described by the differential equations

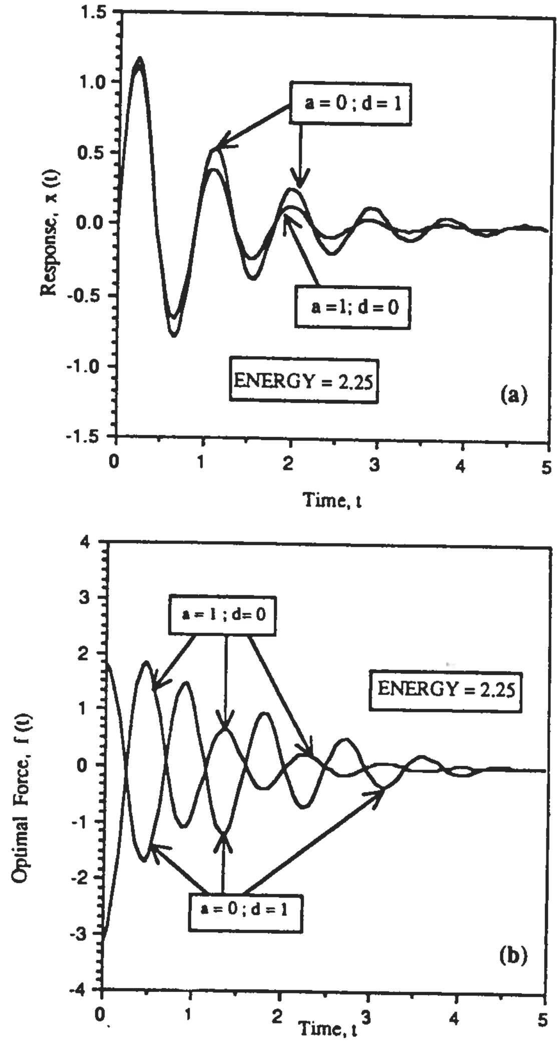


Fig. 1 (a) Comparison of responses for optimal identification (a = 0, d = 1) and minimum response (a = 1, d = 0) for E = 2.25; (b) comparison of optimal forces for optimal identification (a = 0, d = 1) and minimum response (a = 1, d = 0) for E = 2.25.

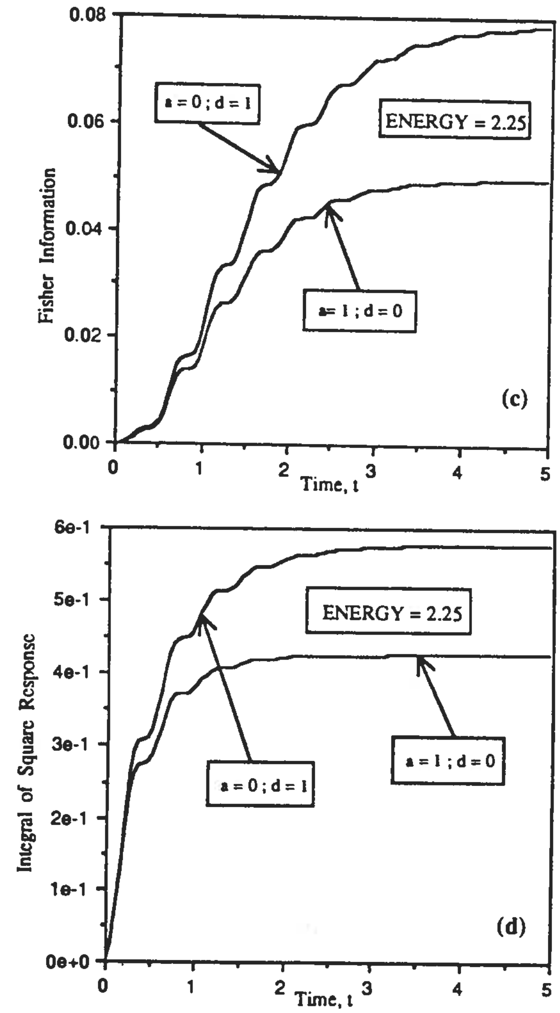


Fig. 1 (c) (Comparison of Fisher information matrix $J_c^{(T)}$ for optimal identification (a = 0, d = 1) and minimum response (a = 1, d = 0) for E = 2.25; (d) Comparison of integral of square response for optimal identification (a = 0, d = 1) and minimum response (a = 1, d = 0) for E = 2.25; (d) Comparison of integral of square response for optimal identification (a = 0, d = 1) and minimum response (a = 1, d = 0) for E = 2.25.

$$\frac{d}{dt}\begin{bmatrix} \mathbf{x}_1 \\ \mathbf{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -k & -c \end{bmatrix}\begin{bmatrix} \mathbf{x}_1 \\ \mathbf{x}_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} \mathbf{f}(t);\ \mathbf{x}_1(0)=a_0,\ \mathbf{x}_2(0)=b_0 \tag{27}$$

where $\mathbf{f}(t)$ is the optimal input to be applied. Denoting $\mathbf{x}=[\mathbf{x}_1\ \mathbf{x}_2]^T$, $\mathbf{x}_k = \partial\mathbf{x}/\partial k$, and $\mathbf{x}_c = \partial\mathbf{x}/\partial c$, the objective function is taken to be

$$J(T) = -a\, J_f(T) + b\, J_k(T) + d\, J_c(T) \tag{28}$$

where,

$$J_f(T) = \int_0^T \mathbf{x}_1^2\, dt, \tag{29a}$$

$$J_k(T) = \int_0^T \mathbf{x}_k^2\, dt, \tag{29b}$$

and,

$$J_c(T) = \int_0^T \mathbf{x}_c^2\, dt. \tag{29c}$$

The weighting factors a, b, and d are taken to be non-negative. This may be thought of as being produced by choosing $\alpha = a$, $\beta = 1$, $\mathbf{W} = \mathrm{Diag}\,(b, d)$, $\mathbf{Q}_1 = \mathrm{Diag}\,(1, 0)$ and $\mathbf{H}_1 = [1\ 0]$ and the scalar $\mathbf{R}_1 = \sigma^2 = 1$ in our general formulation of equations (10)–(18). The six component vector $\mathbf{y}$ is then given by

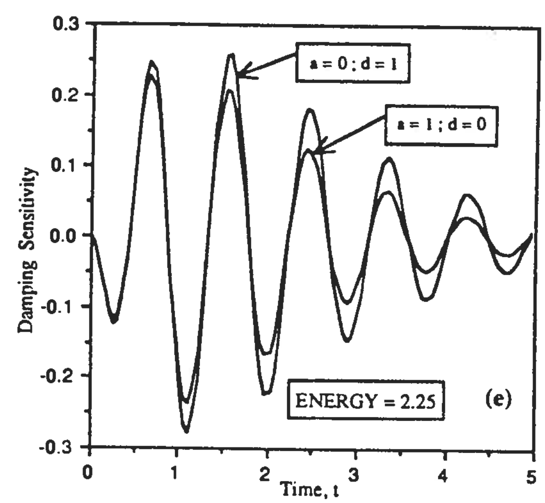
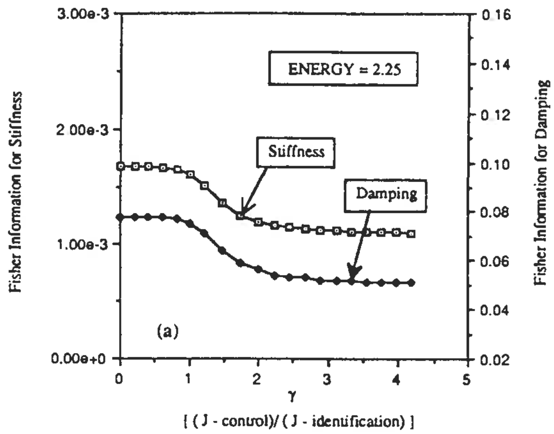


Fig. 1 (e) Comparison of damping sensitivity for optimal identification (a = 0, d = 1) and minimum response (a = 1, d = 0) for E = 2.25

$$y = \begin{bmatrix} \mathbf{x} \\ b^{1/2}\mathbf{x}_k, \\ d^{1/2}\mathbf{x}_c \end{bmatrix}, \tag{30}$$

and the matrix $\mathbf{F}$ becomes



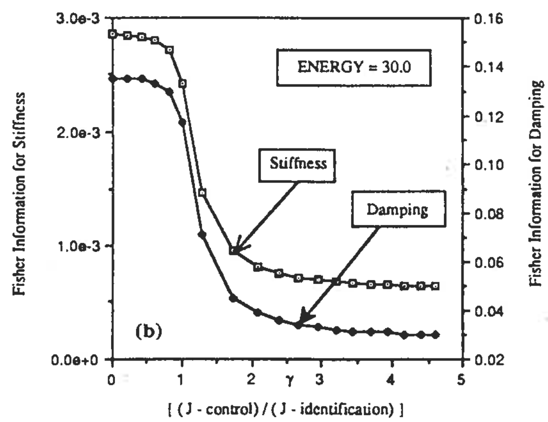


Fig. 2 (a) Fisher information matrices $J_c(T)$ and $J_k(T)$ as function of γ for $E=2.25$; (b) Fisher information matrices $J_c(T)$ and $J_k(T)$ as function of γ for $E = 30.00$.

$$\mathbf{F}=\begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ -k & -c & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ -b^{1/2} & 0 & -k & -c & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & -d^{1/2} & 0 & 0 & -k & -c \end{bmatrix}. \tag{31}$$

Since the vector $\mathbf{G} = [0\ 1\ 0\ 0\ 0\ 0]$, the vector $\{\mathbf{V}\lambda/\eta\}$ has only one nonzero component, namely, $\lambda_2(t)/\eta$, where $\lambda_2(t)$ is the second element of the vector $\lambda(t)$. Figure 1 shows some of the numerical results for the following parameters values (which we shall assume are taken in consistent units):

$$b=0,\ T=5,\ E=2.25;\ \text{and}, \tag{32a}$$

$$k=50,\ c=2,\ \mathbf{x}_1(0)=0,\ \mathbf{x}_2(0)=10. \tag{32b}$$

These parameters thus look at a single-degree-of-freedom oscillator whose spring constant is 50 units, and whose viscous damping is 2 units. This yields a system which has an undamped natural frequency of vibration of about 7 radians/sec and a percentage of critical damping of about 15 percent. It is subjected to an initial velocity of 10 units. The aim is to study the trade-off between (1) identifying (in equation (27)) the damping parameter, c, in the best possible way, and (2) controlling the system so that its mean square response over the time period T is a minimum, given that an input (forcing function) of 5 units duration with an energy of up to 2.25 units is to be used. The two point boundary value problem posed in equation set (22a) is numerically solved using the standard multiple shooting technique (Roberts and Shipman (1972)). The local error tolerance during integration of the differential equations and the permitted error in the satisfaction of the boundary conditions are each set to 10^{-4}. The responses of the system together with the optimal inputs as obtained from equation (23a) are shown for the two extreme cases: (1) $a=0$, $d=1$, corresponding to the optimal input required for identification of the damping parameter c, and (2) $a=1$, $d=0$, corresponding to the optimal input required for minimizing the response. As seen from Fig. 1(b), the optimal inputs required for "best" identification and for "best" control (the term "best" is used in terms of the cost function (28) utilized) are widely different from each other. In fact they are seen to be, for the entire duration over which they last, almost exactly out of phase. Differences in the response of the system to combined influence of the initial velocity and the forcing functions obtained for the two cases are shown in Fig. 1(a). Figure 1(c) shows the Fisher Information Matrices for damping, which in this case are scalars, namely $J_c(t)$, for the abovementioned two extreme cases, as a function of time, t. The difference between these at $T = 5$ is about 55 percent. Alternatively put, the input forcing function, which controls the system response maximally, causes a response which is only about 55 percent as informative about the system parameter c as that caused by a forcing function that is designed to maximally provide information about the parameter c. The manner in which the integral of the response quantity squared, $J_f(t)$, changes with time for the two cases mentioned above is shown in Fig. 1(d). As seen at $T=5$, the optimal control input is about 35 percent more effective in reducing the mean square response than the input which optimally determines the parameter c. Figure 1(e) provides the sensitivity of the response to the damping parameter (at $c=2$) as a function of time.

Figure 2(a) shows the manner in which the Fisher Information matrices $J_c(T)$ and $J_k(T)$ change for various values of the ratio, $\gamma=[\{a\ J_f(T)\}/\{d\ J_c(T)\}]$ when the Fisher values are normalized to unity. It is to be noted that the optimal input when $a=0$ corresponds to that required for "best" estimation of the parameter c in equation (27). Figure 2(b) shows the effect of changing the available control energy, E, from a value of 2.25 to 30 keeping all other parameters the same. From a loss of information in the parameter c of 55 percent in the case of $E = 2.25$, the loss in information when $E = 30$ jumps to about 450 percent. Similarly, the extent to which the system's performance can be controlled deteriorates by about a factor of 3 if one aims purely at identification instead of control. The parameter η which is calculated for each objective function ratio, γ, is shown in Fig. 3. As mentioned in the formulation, this quantity is simultaneously solved for, in the set (22a), thereby eliminating the need to find its value by trial and error. Had this not been done, a very high computational expense would have been incurred to ensure that the energy constraint is satisfied. Noting that the inverse of $J_c(T)$, for an efficient unbiased estimator, is the covariance of the estimate of the parameter c, Figs. 4(a) and 4(b) provide the trade-off between control and identification. As seen in Fig. 4(a), in going from $\gamma=0$ to $\gamma=4$, $J_f(T)$, the mean square response, *falls off* by about 35 percent, similarly, the covariance of the estimate of c *increases* by about 55 percent as γ varies over the same interval. For larger values of the input energy, Fig. 4(b) shows that significant reductions in J_f and significant increases in the covariance of the parameter estimates can occur.

Conclusions and Discussion

In this paper we have presented an approach to quantifying the trade-off between the tasks of control law development and plant identification. To the best of our knowledge this trade-off has never been analyzed quantiatively before. The problem is formulated in the context of optimal control and optimal identification through the intermediary concept of an optimal input. A suitable objective function is chosen so that

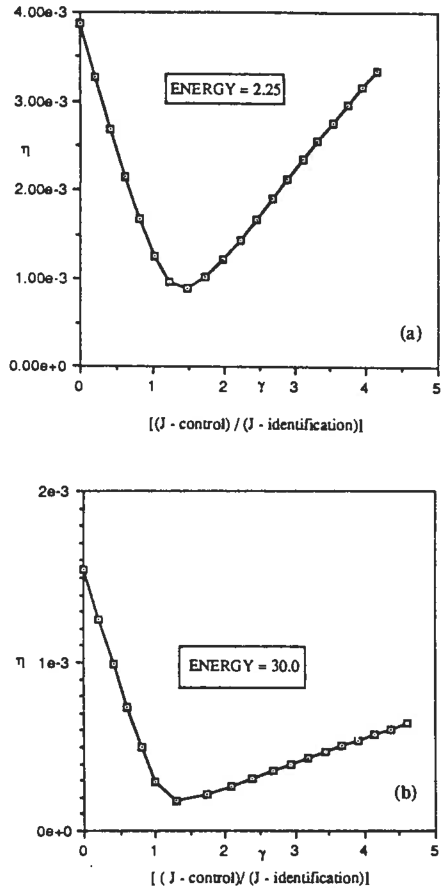


Fig. 3 (*a*) The parameter η as function of γ for E = 2.25; (*b*) the parameter η as function of γ for E = 30.00.

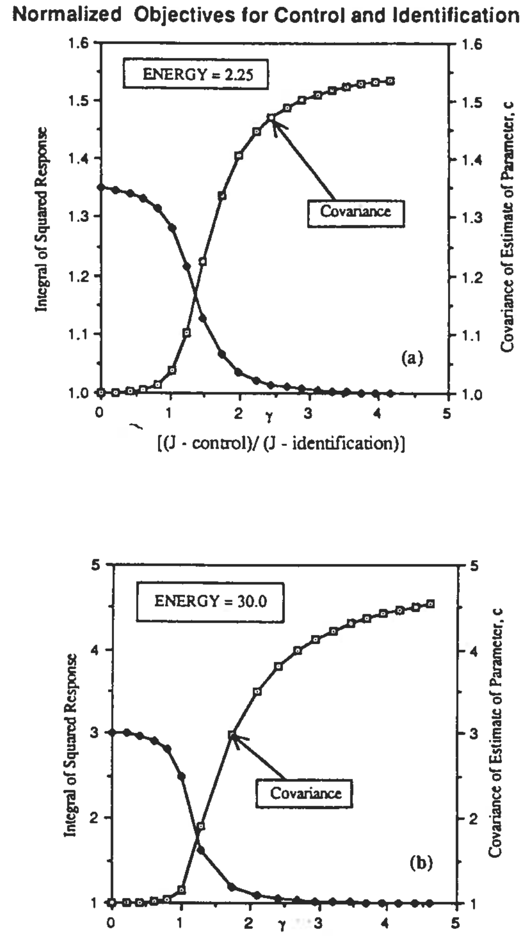


Fig. 4 (*a*) Integral of squared response as function of γ for E = 2.25; (*b*) Integral of squared response as function of γ for E = 30.00.

the emphasis from control to identification can be changed in a continuous manner. It is shown that the duality between identification and control can be quantified by determining optimal inputs, which have a specified amount of energy, and which minimize the objective function. Augmenting the state by an additional variable allows simultaneous solution of the optimization problem together with the energy constraint. Using variational calculus this leads to a two point boundary value problem that is nonlinear due to the introduction of the energy constraint. The boundary value problem is solved numerically using the multiple shooting technique and Newton-Raphson iterations.

A numerical example, which deals with control and identification of the parameters of a single-degree-of-freedom oscillator, is used to illustrate the concepts involved. It is shown that improved control leads to serious deterioration in the covariance of the parameter estimation and vice versa. In general, as the energy of the input increases the trade-offs between identification and control are shown to become more and more intense.

The above example shows the potential of the approach introduced in this paper. The numerical computations can be easily generalized to vibratory systems with many degrees-of-freedom, making the method presented here useful in various fields, like control and identification of large flexible structures. A study of the trade-off between identification and control for systems that follow a prescribed input is currently underway.

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