Pulse Control of Single Degree-of-Freedom System

By F. E. Udwadia[1] and S. Tabaie[2]

Introduction

In the last decade or two, an increasing amount of attention has been given by scientists and engineers all over the world to improving the methodologies for ensuring the safety of structures subjected to various types of dynamic loads. A significant portion of this effort has been devoted to improvements in the areas of structural analysis, design, and construction. Though these advances will no doubt lead to safer designs, in many situations the structural designer is still left with two basic sources of uncertainty: (1) The uncertainty in the time history of loading; and (2) the uncertainty in the dynamic modeling of the system.

An alternative to the exclusive reliance on analytical techniques, the results of which are in turn dependent of these aforementioned uncertainties, is to investigate the possibility of using active control for structural and mechanical systems. It is with this alternative approach that this paper concerns itself.

The concept of active control has attracted considerable interest from the research community in recent years. Various researchers have investigated the use of modern control techniques in the control of structural systems. A collection of recent advances in this area can be found in (4). The work of Abdel-Rohman et al., (1,2), Yang (10), Soon (5), and Leipholtz (4) are but a few examples of feasibility studies using optimal control theoretic methods.

Along with these developments, heuristic algorithms for the active control of structures have also been developed. Sae-Ung and Yao (6) were among the first to use such methods for the active "comfort control" of tall structures subjected to wind loads. They attempted to keep the induced acceleration levels below those that would cause human discomfort, while preventing large storey drifts. They postulated a heavyside step function type of control force and used an empirically obtained control law. However, their method yielded highly

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nonlinear stochastic differential equations causing them to resort to Monte Carlo simulations.

To overcome some of the difficulties mentioned in the preceding paragraph, this paper investigates the feasibility of the concept of open loop adaptive control of structural systems subjected to both deterministic and stochastic excitations. The algorithm, which is developed for a single degree-of-freedom oscillator, is heuristic in nature and requires a continuous monitoring of the state variables. Following the determination that some prespecified threshold has been exceeded, an open loop pulse control is applied. The use of pulses of short duration to control the system is motivated by the frequently encountered difficulty of applying large control forces over sustained periods of time. The determination of the optimum pulse magnitude is based on the minimization of the root mean square response of the system. The simplified cost function used, enables a closed form solution for the pulse magnitude, thereby considerably reducing the on-line computational effort. The technique is applied through simulation to both linear time invariant and time variant single degree-of-freedom systems. An example for a nonlinear system is also provided.

Control Algorithm

Consider a structural or mechanical system modelled as a single degree of freedom (SDOF) oscillator with mass M, and restoring force, $F = F(x, \dot{x}, t)$, in which x and $\dot{x}$ are the displacement and velocity of the oscillator relative to its base. Let $f(t)$ be the dynamic load applied to the mass M, and $\ddot{z}(t)$ be the base acceleration (Fig. 1). The equation of motion is then

$$M\ddot{x} + F(x, \dot{x}, t) = -M(\ddot{z}_s + \ddot{z}_d) + f_s(t) + f_d(t) \quad \ldots\ldots\ldots\ldots\ldots \quad (1)$$

in which the subscripts s and d denote the stochastic and deterministic components of $\ddot{z}$ and f, respectively.

Control Algorithm.—As the control algorithm is heuristic in nature, for purposes of development, we shall begin by assuming that the system is linear with $F = Kx + C\dot{x}$, in which K and C are the constant stiffness and viscous damping values, respectively (Fig. 1(a)).

The heart of the control algorithm lies in the physical realization that the gradual rhythmic build up of the structural response (vibrational energy) can be destroyed by applying a pulse of suitable magnitude in the proper direction. To minimize the amount of control energy, the control should be applied only when the response amplitude exceeds a certain threshold value, x_T, related to the resistance of the structure. Furthermore, as this build-up of motion has a characteristic time of the order of the natural period, of the system, T, the minimum spacing, T_p, between the pulses should be taken to be at least of $O(T)$ (Fig 1(c)).

The time duration of each pulse, t_w, as well as T_p primarily would be controlled by the response time of the pulsing equipment used. It will be assumed, for simplicity, that rectangular pulse forms will be generated, though of course other pulse forms can be just as easily used.

We shall require that the pulse magnitude, P_o, be so chosen as to minimize the mean square response of the oscillator in the interpulse interval, under the constraint that the pulse not exceed a certain predetermined value, P_u.

The value P_u would again depend on the limitations of the pulsing equipment. Then, at time t_o (Fig. 1(*c*)), the oscillator velocity, $\dot{x}_o$, and displacement, x_o, are known, and an optimal pulse height must be ascertained. The pulse height, P_o, requires the minimization of the functional

$$J = \frac{1}{T_p}\int_{t_o}^{t_o+T_p} x^2(t)\,dt \qquad (2)$$

under the constraint $P_L \le P_o \le P_u$, where P_L is the lower bound amplitude for the pulse. The response, $x(t)$, may be thought of as being caused by: (1)

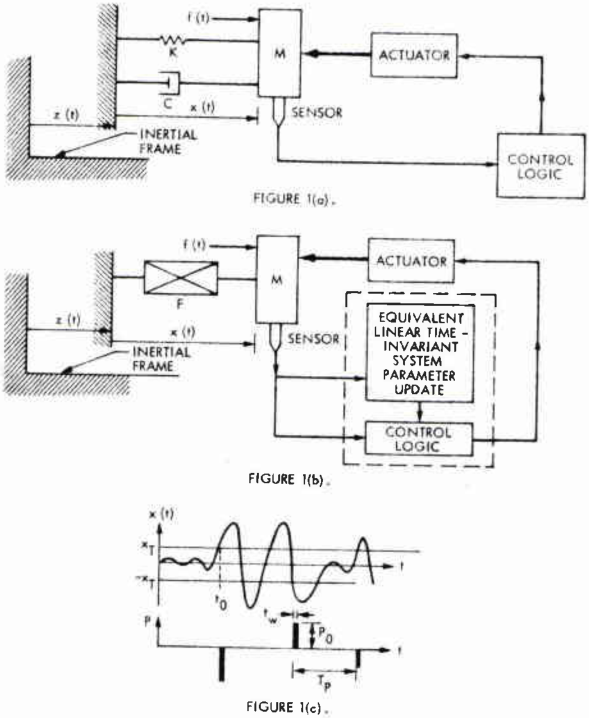

FIG. 1.—Conceptual Design of Open Loop Adaptive Pulse Control for Linear (*a*) and Nonlinear (*b*) Systems

The initial conditions at time t_o; (2) the application of the pulse, P_o, at time t_o (of width t_w); (3) the stochastic components of the dynamic load and the base motion; and (4) the deterministic components of the dynamic load and base motion. The four contributions can be expressed as x_i, x_p, x_s and x_d, respectively, so that for the linear system

$$x(t) \stackrel{\Delta}{=} x_i(t) + x_p(t) + x_s(t) + x_d(t) \qquad (3)$$

Since the pulse is determinisitic and is applied at time t_o, and since at time t_o the time histories of the stochastic components are unknown for the interval

$(t_o, t_o + T_p)$, the pulse height, P_o, cannot be designed to control the part of the motion created by the stochastic loads and base excitations which occur in $(t_o, t_o + T_p)$. Though the pulse at time t_o would, in a sense, be cognizant of the excitation (created by the external dynamic environment as well as the pulse control forces) and the response of the system prior to time t_o, through the initial conditions at time t_o, it cannot be tailored to explicitly take into account the stochastic components in the interpulse interval. However, if t_w and T_p are small (t_w/T, $T_p/T << 1$), then one can further simplify the cost function, J, by ignoring the stochastic components that occur in the interpulse interval. This assumes that the vibrational energy of the oscillator at time t_o is relatively large compared with the increment in energy which it would have acquired from the additional stochastic components occuring in the time interval $(t_o, t_o + T_p)$. If $P(t)$ is the control force time history, then the response, $x(t)$, can now be written as

$$x(t) = x_i(t) + x_p(t) + x_d(t), \quad t \in (t_o, t_o + T_p) \quad \ldots \quad (4)$$

$$= x_o u(t - t_o) + \dot{x}_o v(t - t_o) + \int_{t_o}^{t} h(t - \tau) \frac{P(\tau) d\tau}{M} + x_d(t) \quad \ldots \quad (5)$$

in which $u(t) = \exp(-\omega_n \zeta t)$ (Cos $\omega_d t + \omega_n \zeta/\omega_d$ Sin $\omega_d t$); $v(t) = 1/\omega_d$ exp $(-\omega_n \zeta t)$ Sin $\omega_d t$; $h(t) = v(t)$; $\omega_d = \omega_n \sqrt{1 - \zeta^2}$, with $\omega_n = \sqrt{K/M}$; and $\zeta = C/(2\sqrt{KM})$.

Furthermore, if $P(t)$ is restricted to the class of pulses of duration t_w with amplitudes P_o, then

$$x(t) = x_o u(t - t_o) + \dot{x}_o v(t - t_o) + \int_{t_o}^{t_o + t_w} \frac{1}{M} P_o h(t - \tau) d\tau + x_d(t),$$

$$t_o + t_w \leq t \leq t_o + T_p \quad \ldots \quad (6)$$

Also, if $(t_w/T) << 1$, then

$$x(t) = x_o u(t - t_o) + \left(\dot{x}_o + \frac{I}{M}\right) v(t - t_o) + x_d(t), \quad t_o \leq t \leq t_o + T_p \quad \ldots \quad (7)$$

in which I = the impulse created by the pulse, of magnitude $P_o t_w$. Denoting $I = P'_o \delta(t - t_o)$, we have

$$J(P'_o) = \int_{t_o}^{t_o + T_p} \left\{ \left[x_o \text{Cos}\, \omega_d \tilde{t} + \frac{1}{\omega_d} \left(\dot{x}_o + x_o \omega_n \zeta + \frac{P'_o}{M} \right) \text{Sin}\, \omega_d \tilde{t} \right] e^{-\omega_n \zeta \tilde{t}} + x_d(t) \right\}^2 dt \quad \ldots \quad (8)$$

in which $\tilde{t} = t - t_o$.

Differentiating with respect to P'_o and setting the derivative to zero, we have

$$P'_o = -\left(\frac{Y_1 + Y_d}{Y_2 + Y_3} + A \right) M \quad \ldots \quad (9)$$

in which Y_1, Y_2, Y_3, and A are evaluated in Appendix I. Using t_w as the pulse duration, the pulse height, P_o, can be approximated as

$$P_o \simeq \frac{P'_o}{t_w} \quad \text{(10)}$$

Eqs. 4–10, though representing approximate solutions, yield results which can be very efficiently implemented for on-line computations.

In the absence of deterministic inputs, $Y_d = 0$; also, for a given time-invariant linear system, Y_1, Y_2, and Y_3 are constants for a given set of control parameters t_w and T_p. Thus, they can be calculated off-line. The only on-line computations

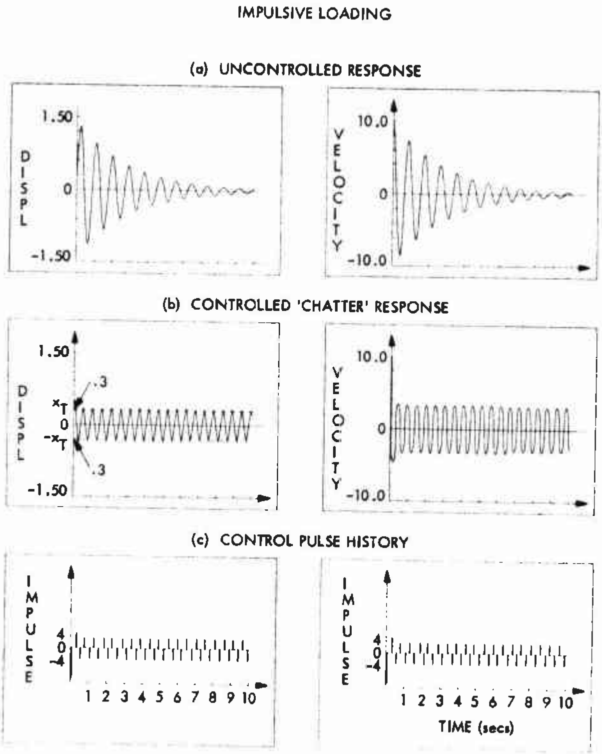

FIG. 2.—Control for Impulse Load Showing Chatter Phenomenon

that then would be required in that case are those given by Eqs. 9 and 10. Thus, the computational requirements become negligible when compared with those required for optimal continuous feedback control.

Chatter Suppression.—Fig. 2(*b*) shows the nature of the controlled response, obtained by this technique, of an oscillator when subjected to an impulse at zero time of magnitude ten units, $K = 50$, $M = 1$, $\zeta = 5\%$, $T \cong 1.1$ sec. The parameters t_w, T_p, and x_T were set to 0.04 sec, 0.2 sec, and 0.3 length units, respectively. We observe (Figs. 2(*a*) and 2(*b*)) that while the pulse control quickly (within almost one cycle) brings the amplitude level of the response to the threshold amplitude, x_T, the motions of the oscillator are essentially perpetuated beyond that time by the alternate pulsing done by the control, despite the fact that the forcing function is identically zero then. The controlled response, though within the threshold limits, will then exceed the uncontrolled response at large times, depending on the specific values of ζ and x_T involved. The "chatter phenomenon," which commonly occurs in 'bang-bang' types of control systems, is caused by the periodic pulsing (Fig. 2(*c*)) of the oscillator so that the amplitude levels of response always lie within the threshold limits. Ideally, one would want to design the control system so that after the preassigned threshold response is achieved, the pulse control is cut off, allowing the motions to die down by virture of the damping that is present in the system.

To achieve this end, the response of the system is tracked, and when the system goes into a chatter mode of vibration, the pulse amplitude is set to zero. This sudden drop in pulse amplitude generally will cause a slight overshoot

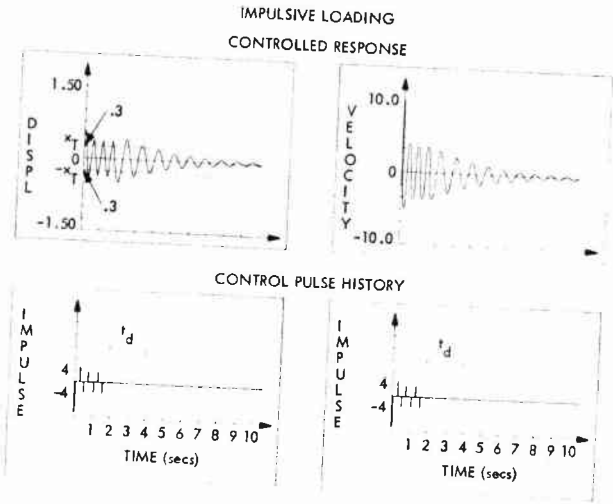

FIG. 3.—Control for Impulsive Response with Chatter Suppression

in the system response (beyond the level x_T) to occur. If this overshoot is less than α times x_T in which $\alpha > 1$ is a preassigned constant, then the pulsing is stopped for a period of time, t_d. However, if a large overshoot (which could conceivably be caused by stochastic excitations occuring within the time period t_d) occurs, then the pulsing is continued in abeyance of the 'dead-time,' t_d. The values of t_d and α, to be chosen for the control logic, depend on the system characteristics, x_T, and the allowable overshoot beyond x_T that is deemed nondamaging to the structure. In practice, of course, the x_T can be chosen sufficiently small so that adequate control is achieved.

Fig. 3 shows the implementation of the aforementioned control logic with $t_d = 2.0$ sec and $\alpha = 1.5$. At about 1.8 sec, the chatter mode is recognized by the tracking algorithm and the pulse amplitude is set to zero. This yields a slight overshoot beyond x_T and the system from there on is left to come to rest without any further control.

Nonlinear and Time Varying Systems.—The control methodology so far developed can be applied to nonlinear and time-variant systems if, over periods of time T_o of $O(T_p)$, the equivalent time-invariant linear system properties are obtained. Perhaps the easiest way of performing this for SDOF systems is by means of a moving window fourier analysis (8). Such an update would involve an increased on-line computational load (7). Thus, the elimination of this update procedure, whenever possible, would help in reducing the computational requirements, especially for multidegree of freedom systems. Most structural systems, on entering a strongly nonlinear range of response, often undergo partially damaging vibrations. Thus, if the values of x_T are fixed so that the response remains primarily in the linear (nondamaging) regime, the necessity of incorporating equivalent linearization in the control strategy may be obviated, thereby minimizing the on-line computational effort. The flow chart for the openloop logic is shown in Fig. 4.

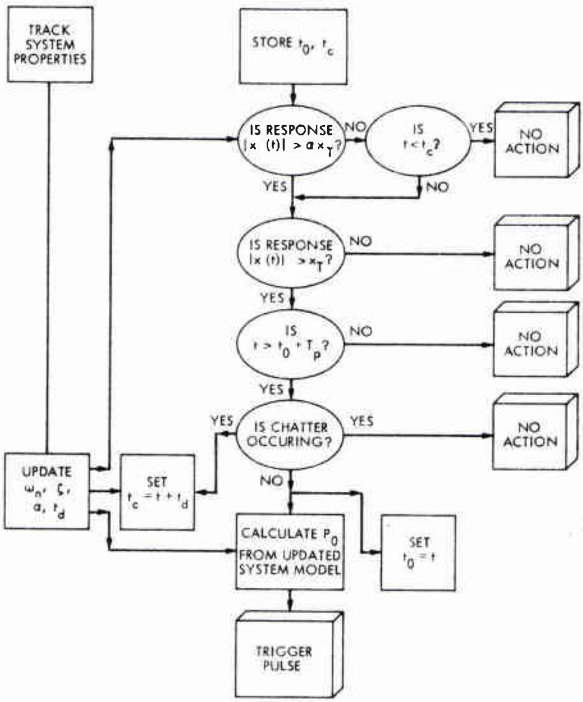

FIG. 4.—Flow Chart for Control Logic

Simulation of Control Strategy and Applications

Fig. 5(c) shows an artificial accelerogram generated by using a time modulated white noise signal of the form

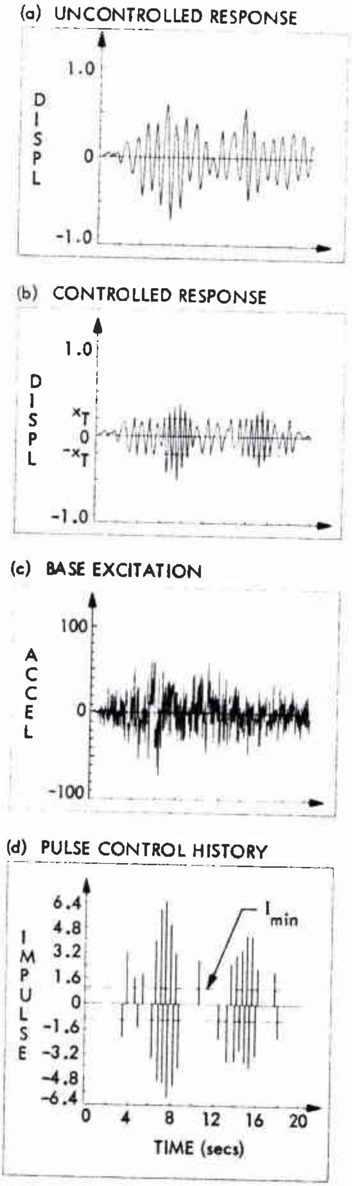

FIG. 5.—Uncontrolled (*a*) and Controlled (*b*) Responses to Base Excitation (*c*) for Linear Time-Invariant System

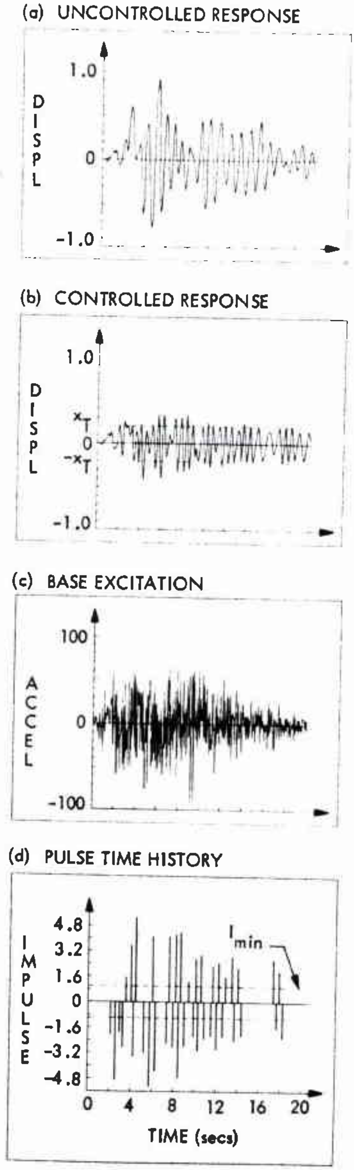

FIG. 6.—Uncontrolled (*a*) and Controlled (*b*) Responses to Base Excitation (*c*) for Linear Time-Invariant System

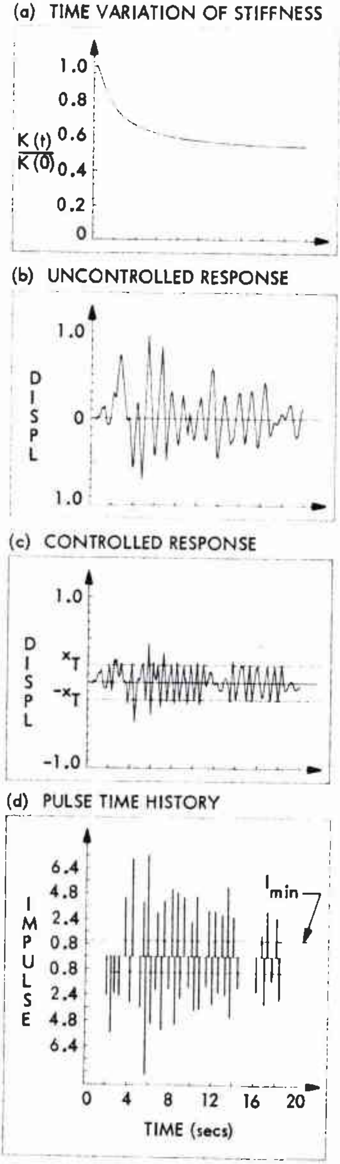

FIG. 7.—Uncontrolled (*a*) and Controlled (*b*) Response of Linear Time-Variant System to Base Excitation on Fig. 6(*c*)

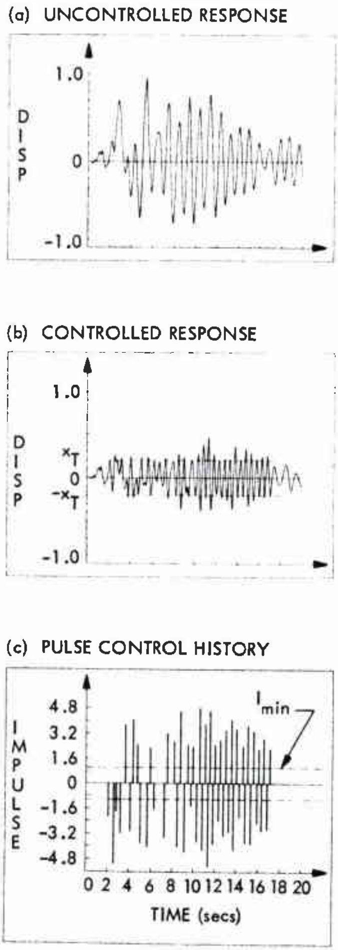

FIG. 8.—Uncontrolled (*a*) and Controlled Response (*b*) of Nonlinear Softening System to Base Excitation of Fig. 6(*c*)

$$\ddot{z}(t) = a_1 t e^{-\beta t} n(t) \qquad (11)$$

in which $n(t)$ is a sample of zero mean gaussian white noise (ZMGWN) with variance equal to unity; and in which a_1 and β are constant parameters. A structural system modelled as an SDOF oscillator (with $M = 1$, $K = 50$, $\zeta = 5\%$, and $T \simeq 1.1$ sec) is subjected to this stochastic base excitation using $a_1 = 10$ and $\beta = 0.15$ sec^{-1}. Twenty seconds of response of the oscillator are shown in Fig. 5(*a*).

The control strategy described in the previous section is implemented and the controlled response of the system is shown together with the time history of the impulses, I, used for the control in Figs. 5(*b*) and 5(*d*). The control parameters chosen are $t_w = 0.04$ sec, $T_p = 0.2$ sec, $x_r = 0.2$ length units, $\alpha = 1.5$, $t_d = 2.0$ sec, $I_{min} = 1$, and $I_{max} = 10$. For each simulation reported in this sequel, to take account of delays due to the actuator response time, the pulse magnitude calculated at time t_n is applied to the system after a delay of 0.02 sec. Fig. 6 shows the efficacy of the control methodology (using the same control parameters) with a different stochastic excitation which is generated using a different sample of ZMGWN, with $a_1 = 20.0$ and $\beta = 0.2$ sec^{-1}.

A time variant linear degrading system ($M = 1$), the stiffness of which $K(t)$, changes according to the relation (3,7,8)

$$K(t) = 0.5 K_o \left[2 - \exp\left(-\frac{\delta}{t} \right) \right] \qquad (12)$$

in which $K_o = K(t = o) = 50$ and $\delta = 1.5$ sec, is depicted in Fig. 7(*a*). The stochastic base excitation is taken to be identical to that of Fig. 6(*c*). The uncontrolled response of the system is shown in Fig. 7(*b*). To study the sensitivity of the control technique to knowledge of the updated system properties, the control algorithm is implemented with the same control parameters as before *without* making any system property updates. The system parameters in the control logic are assumed to be time-invariant and equal in value to those at zero time. As seen in Fig. 7(*c*), the control strategy works reasonably well in limiting the displacement response to lie between $\pm x_r$.

Lastly, we consider a nonlinear 'softening' system subjected to the base acceleration of Fig. 6(*c*). The system properties are described by

$$F(x, \dot{x}) = 50x - 30x^3 + 0.705\dot{x}, \quad \text{and} \quad M = 1. \qquad (13)$$

The uncontrolled response of the system is shown in Fig. 8(*b*). Again, the control strategy is implemented without taking heed of the nonlinearity of the system and without system parameter updates. The same control parameters as before are used and the system, for the control logic, is assumed linear (with $K = 50$, $M = 1$, $\zeta = 5\%$). The controlled response (Fig. 8(*c*)) shows that the control strategy is fairly effective even *without* tracking the equivalent linearized system properties. As mentioned earlier, such tracking could increase the online computational load considerably, if required.

Analysis and Conclusions

This paper shows the feasibility of using on-line, pulsed, open-loop adaptive control for reducing the oscillations of structural and mechanical systems modeled as SDOF oscillators, subjected to dynamic load environments. The adaptive feature is incorporated to take into account the nonlinear, time variant nature of such systems; the pulse control is incorporated to get around our inability to produce large control forces over sustained periods of time; and, the open loop configuration is incorporated to reduce online computation times. The method differs from optimal control theoretic methods in that it is heuristic in nature. It attempts to formulate a control methodology which, while perhaps being suboptimal, is found to be more than adequate in the many simulation studies conducted to date. The sensitivity of the performance of the control technique to measurement noise ($N/S \sim 1/50$) appears to be very low in the simulation studies done so far. This principally is due to the adaptive nature of the control strategy. Whereas this paper does not investigate the stability of the control method, preliminary results show that under proper choices of the control parameters, the controlled response is asymptotically stable (9).

The control algorithm essentially requires: (1) A continuous tracking of the system properties to obtain the updated equivalent linear, time-invariant, system parameters; and (2) a continuous monitoring of the system state to determine if a specified threshold displacement is exceeded. No assumptions about the nature of the stochastic excitation have been made. However, it is assumed that the vibrational energy imparted to the system during each interpulse interval is small compared with the total energy of the system at the beginning of that interval. The actual computations for determining the pulse magnitude, which are carried out in the time domain, are shown to be quick and easy to perform once the updated system parameters are available.

As the aforementioned first requirement poses a considerable on-line computational job, especially for multidegree of freedom systems, the sensitivity of the control strategy to knowledge of the updated parameter values is studied. It is found that as long as the time variations are not excessive, and as long as the nonlinear system response is controlled to lie within a threshold amplitude range in which the system is not strongly nonlinear, the parameter update may not be needed. Extensions of the method to incorporate threshold levels related to acceleration and velocity, or combinations thereof are easy to implement. The method has been applied to multidegree of freedom systems and shows considerable promise. These topics will be addressed in a future communication.

The primary asset of this method lies in its basic simplicity, a factor which foreshadows the reliable functioning of any on-line control system.

Acknowledgment

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Appendix I.—Additional Formulations

Denoting T_p by T, the following relationships are established:

$$\left.\begin{aligned}
Y_d &= \int_{t_o}^{t_o+T} 2\,x_o(t)\exp(-\omega_n\zeta\tilde{t})\,\mathrm{Sin}\,\omega_d\tilde{t}\,dt;\\
Y_1 &= x_o\frac{[e^{S_1T}(S_1\,\mathrm{Sin}\,S_2T - S_2\,\mathrm{Cos}\,S_2T) + S_2]}{D};\\
Y_2 &= \frac{1}{2\omega_n\zeta\omega_d}[1-\exp(-2\omega_n\zeta T)]\\
Y_3 &= \frac{\{S_1 - \exp(S_1T)[S_1\,\mathrm{Cos}\,S_2T + S_2\,\mathrm{Sin}\,S_2T]\}}{\omega_d D};\\
A &= (\dot{x}_o + \omega_n\zeta x_o);
\end{aligned}\right\} \qquad (14)$$

in which $S_1 = -2\omega_n\zeta$; $S_2 = 2\omega_d$; and $D = S_1^2 + S_2^2$.

Appendix II.—References

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Appendix III.—Notation

The following symbols are used in this paper:

a_1 = constant parameter;
C = viscous damping;
F = restoring force;
$f(t)$ = applied load;

f_d = deterministic component of applied force;
f_s = stochastic component of applied force;
I = impulse created by pulse;
I_{max} = maximum impulse;
I_{min} = minimum impulse;
J = functional denoting mean square response of oscillator;
K = system stiffness;
K_o = stiffness at zero time;
$K(t)$ = time varying stiffness;
M = system mass;
$n(t)$ = Gaussian white noise;
P_o = pulse magnitude;
P_L, P_u = lower and upper bonds per pulse amplitude;
T = system period;
T_p = interpulse interval;
t_w = pulse duration;
t_d = dead-time;
x = relative displacement;
x_d = displacement caused by deterministic load;
x_i = initial displacement;
x_p = displacement caused by pulse;
x_s = displacement caused by stochastic input;
x_t = response amplitude threshold;
$\ddot{z}$ = base acceleration;
$\ddot{z}_d, \ddot{z}_s$ = deterministic and stochastic component of base acceleration;
α = constant parameter;
β = constant parameter;
δ = time parameter;
ω_n = undamped natural frequency;
ω_d = damped natural frequency; and
ζ = percentage of critical damping.