Pulse Control of Structural and Mechanical Systems

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Introduction

The concept of active control of flexible structural and mechanical systems is one which has received a considerable amount of attention from the research community in recent years. The difficulties of predicting future loading time histories on the basis of limited amounts of available past records, the uncertainties in establishing material property characteristics, and the lack of knowledge of the proper mathematical models involved in making response predictions are three of the main factors that have contributed to making this approach an attractive alternative for increasing the safety of structural and mechanical systems.

Several control methods have been investigated by various researchers. Ref. 3 contains a large number of recent contributions to the field. For instance, Yao and Tang (7) have considered the application of a series of heavyside step functions for the control of structures for providing appropriate human comfort in tall buildings. They decided on the control force history, for stochastic loading conditions, by means of a trial and error procedure. The feasibility of using modal control (4), the pole assignment method (1), and optimal stochastic control (2) are but a few examples of still other approaches that have been studied. Modern control theoretic methods, however, typically lead to feedback control laws which require: (1) The specification of weighting matrices (generally done by a trial and error process); and (2) the solution of the necessary Riccati equations that arise in the determination of the control force time history. Though these requirements may not be unreasonable for systems with a small number of degrees-of-freedom, the on-line solution of the matrix Riccati equations may become computationally unfeasible for large multidegree-of-freedom systems.

This paper investigates the use of open loop adaptive pulse control for limiting the response of large linear multidegree-of-freedom systems. The control algorithm closely follows that developed in (6) for single degree-of-freedom systems.

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The approach has been motivated by the need to: (1) Cut down on the on-line computational effort required for large multidegree-of-freedom systems; (2) circumvent our inability to produce large control forces (which may be necessary to control massive multidegree-of-freedom systems) over sustained periods of time; and (3) develop a control system which is simple to implement thereby enhancing its functional reliability.

The feasibility of the pulse control concept for systems, subjected to both deterministic and stochastic excitations, has been studied. The algorithm which is developed for a linear multidegree-of-freedom system is heuristic in nature and requires a continuous monitoring (or estimation) of the system state. When the system response exceeds some specified threshold, an open loop pulse control is applied at a set of preassigned actuator locations. The determination of the optimum pulse magnitude is based on the minimization of the sum of the weighted Euclidean norms of the velocity and displacement vectors. The cost function used enables a closed form solution for the pulse magnitudes at the various locations, thereby considerably reducing the on line computational effort.

For systems which are linear but time variant, the time dependent system properties need to be tracked. These updated system properties are then adaptively used for determination of the control pulse magnitudes. The technique has been applied through simulation to control the response of a four degree-of-freedom linear system subjected to nonstationary earthquake-like base excitations.

Control Algorithm

Consider a structural or mechanical system modeled as a multidegree-of-freedom system the motion of which relative to the base, $\mathbf{x}$, is described by the equation

$$\mathbf{M}\ddot{\mathbf{x}} + \mathbf{C}\dot{\mathbf{x}} + \mathbf{K}\mathbf{x} = \mathbf{f}_d(t) + \mathbf{f}_s(t) \quad \text{. . . . . (1)}$$

in which $\mathbf{M}$, $\mathbf{C}$, and $\mathbf{K}$ are the $N \times N$ mass, stiffness, and damping matrices, respectively; and $\mathbf{f}_d(t)$ and $\mathbf{f}_s(t)$ are the deterministic and stochastic components of the dynamic loads.

To limit the nodal response of the system, we shall use M actuators and require that the pulses p_{s_i} be applied at nodes s_i, $i = 1, 2, \ldots, M$. For notational convenience, we shall order these nodes so that $s_1 < s_2 < \ldots < s_M$. Furthermore, these pulses, for simplicity of the control logic, when required will be applied at the same time, and will each be characterized by a pulse magnitude, $p^m_{s_i}$, and a fixed characteristic time, t_w, representative of the pulse duration. As the smallest characteristic time for the build-up of the oscillations of the system is of the order of the lowest natural period, T_N, (which would in general correspond to the highest mode which we are interested in controlling), the minimum interpulse spacing, T_p, between the pulses should ideally be $O(T_N)$.

Triggering Criteria.—As the power generation for the creation of the control pulses preferably should be contained within the oscillating system, the economical use of this power supply demands that the pulses be generated only when the system state exceeds a certain threshold level. This threshold would depend on the nature of the system, its projected structural resistance, and the acceptable level of damage. Based on these requirements, one could formulate various triggering criteria.

The triggering criterion used in the sequel is given by the relation

$$a_i|x_i| + b_i|\dot{x}_i| > \eta_i, \quad i = \ell_1, \ell_2, \ldots, \ell_k, \ell_i \in (1, N) \quad \ldots\ldots (2)$$

in which a_i, b_i, and η_i are preassigned constants. Thus, when the system state at time t_o satisfies Eq. 2, an M-dimensional pulse vector, $\mathbf{P} = (p_{s_i})$, is required to be determined to minimize the functional, J, given by

$$J = \frac{1}{2}\int_{t_o}^{t_o+T_p} \mathbf{x}^T\mathbf{Q}_1\mathbf{x}\,dt + \frac{1}{2}\int_{t_o}^{t_o+T_p} \dot{\mathbf{x}}^T\mathbf{Q}_2\dot{\mathbf{x}}\,dt \quad \ldots\ldots (3)$$

in which Q_1 and Q_2 are suitable symmetric, positive definite weighting matrices.

Physical limitations of the pulsing mechanism may further require the imposition of constraints such as

$$p_{s_i}^L < p_{s_i}^m < p_{s_i}^u, \quad i = 1, 2, \ldots, M \quad \ldots\ldots (4)$$

in which $p_{s_i}^L$ and $p_{s_i}^u$ are the lower and upper bound pulse magnitudes at nodes s_i, $i = 1, 2, \ldots, M$.

Modal Decomposition.—The pulses so generated may be thought of as comprising an additional forcing N-vector, $\mathbf{r}$, on the right hand side of Eq. 1, in which

$$\mathbf{r} = \mathbf{SP} \triangleq \begin{matrix} \\ \\ \\ \text{row } s_i \rightarrow \\ \\ \\ \end{matrix} \overset{\text{col } i \;\downarrow}{\begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}} \begin{bmatrix} p_{s_1} \\ p_{s_2} \\ p_{s_3} \\ p_{s_i} \\ \vdots \\ p_{s_M} \end{bmatrix} \quad \ldots\ldots (5)$$

Each row element of the N × M "selection matrix," S, is zero, except for the (s_i, i) element which is unity. Assuming $\mathbf{C} = \alpha\mathbf{M} + \beta\mathbf{K}$, the response, $\mathbf{x}$, can be decomposed in terms of the normal modes as $\mathbf{x} = \phi\mathbf{y}$ in which $\phi^T\mathbf{M}\phi = I$; and $\phi^T\mathbf{K}\phi = (\omega_n^2) \triangleq \Lambda$.

The usual modal decomposition then yields

$$I\ddot{\mathbf{y}} + 2Z_1\dot{\mathbf{y}} + \Lambda\mathbf{y} = \phi^T\mathbf{f}_d(t) + \phi^T\mathbf{f}_s(t) + \phi^T\mathbf{SP}, \; t\in(t_o, t_o + T_p) \quad \ldots\ldots (6)$$

in which $Z_1 = \lceil \omega_n\xi_n \rfloor$; ω_n = undamped natural frequency of the nth mode; $2\xi_n = (\alpha/\omega_n + \beta\omega_n)$; and $\omega_{d,n} = \omega_n\sqrt{1-\xi^2}$.

The modal coordinate, $\mathbf{y}(t)$, may be thought of as created by: (1) The initial conditions $\mathbf{y}(t_o)$ and $\dot{\mathbf{y}}(t_o)$; (2) the application of the pulse, $\mathbf{P}$, at time t_o, of characteristic width t_w; (3) the stochastic component of the dynamic loading; and (4) the deterministic component of the dynamic load. The four contributions may be expressed as $\mathbf{y}_I$, $\mathbf{y}_p$, $\mathbf{y}_s$, and $\mathbf{y}_d$, respectively, so that

$$\mathbf{y}(t) = \mathbf{y}_I + \mathbf{y}_p + \mathbf{y}_s + \mathbf{y}_d \quad \ldots\ldots (7)$$

Since our control pulse, $\mathbf{P}$, is deterministic, and is applied at time t_o, and since at time t_o the stochastic component of the dynamic load is not known for

$t \epsilon (t_o, t_o + T_p)$, the pulse magnitude cannot be designed to control that part of the motion created by the stochastic loading which occurs in the interpulse interval. The pulse, **P**, at time t_o is, however, fully cognizant of the complete dynamic loading history prior to time t_o, through the initial conditions at time t_o.

However, if t_w and T_p are small ($T_p/T_N < 1$), then one may further simplify the cost function, J, by ignoring the stochastic components that occur in the interpulse interval. This assumes that the increment in energy of the oscillating system created by the stochastic excitation in the interpulse interval $t \epsilon (t_o, t_o + T_p)$ is small compared with the energy of the system at time t_o.

The solution of Eq. 6 then yields

$$\mathbf{y}(t) \simeq \mathbf{U}_1(\tilde{t})\,\mathbf{y}(t_o) + \mathbf{V}_1(\tilde{t})\,\dot{\mathbf{y}}(t_o) + \int_{t_o}^{t} V_1(t-\tau)\,\boldsymbol{\phi}^T \mathbf{S}\mathbf{P}(\tau)\,d\tau + \mathbf{y}_d(t),\quad t \epsilon (t_o, t_o + T_p) \quad \ldots \ldots (8)$$

in which $\mathbf{U}_1$ and $\mathbf{V}_1$ are diagonal matrices defined as

$$\left.\begin{aligned} &\mathbf{U}_1(t) = \mathbf{U}(t) + \mathbf{Z}_1\mathbf{V}_1(t), \quad \mathbf{V}_1(t) = \left\lceil \frac{1}{\omega_{d,n}} e^{-\omega_n \xi_n t} \sin \omega_{d,n} t \right\rfloor, \\ &\text{with} \quad \mathbf{U}(t) = \lceil e^{-\omega_n \xi_n t} \cos \omega_{d,n} t \rfloor \quad \text{and} \quad \tilde{t} = t - t_o. \end{aligned}\right\} \quad \ldots \ldots (9)$$

For notational convenience, here we introduce the derivatives with respect to time, t, as

$$\mathbf{U}_2 \triangleq \dot{\mathbf{U}}_1 = \dot{\mathbf{U}} + Z_1\dot{\mathbf{V}}_1 = -\Lambda \mathbf{V}_1, \quad \text{and} \quad \mathbf{V}_2 \triangleq \dot{\mathbf{V}}_1 = \mathbf{U} - Z_1\mathbf{V}_1 \quad \ldots \ldots (10)$$

If the control pulses are rectangular and of width t_w, then Eq. 10 simplifies to

$$\mathbf{y}(t) = \mathbf{U}_1(\tilde{t})\,\mathbf{y}(t_o) + \mathbf{V}_1(\tilde{t})\,\dot{\mathbf{y}}(t_o) + \left[\int_{t_o}^{t_o+t_w} \mathbf{V}_1(t-\tau)\,d\tau\right] \boldsymbol{\phi}^T \mathbf{S}\mathbf{P}^m + \mathbf{y}_d(t),\quad t \epsilon (t_w, t_o + T_p) \quad \ldots \ldots (11)$$

with $\mathbf{P}^m = (p^m_{s_i})$.

Eq. 3 can now be expressed as

$$J = \frac{1}{2}\int_{t_o}^{t_o+T_p} (\mathbf{y}^T \mathbf{A}_1 \mathbf{y} + \dot{\mathbf{y}}^T \mathbf{A}_2 \dot{\mathbf{y}})\,dt \quad \ldots \ldots (12)$$

in which $\mathbf{A}_1 = \boldsymbol{\phi}^T \mathbf{Q}_1 \boldsymbol{\phi}$ and $\mathbf{A}_2 = \boldsymbol{\phi}^T \mathbf{Q}_2 \boldsymbol{\phi}$. Minimization of J, with respect to **P** (we drop the superscript for brevity), requires

$$\frac{\partial J}{\partial \mathbf{P}} = \int_{t_o}^{t_o+T_p} \left(\frac{\partial \mathbf{y}^T}{\partial \mathbf{P}} \mathbf{A}_1 \mathbf{y} + \frac{\partial \dot{\mathbf{y}}^T}{\partial \mathbf{P}} \mathbf{A}_2 \dot{\mathbf{y}}\right) dt = 0 \quad \ldots \ldots (13)$$

in which $(\partial \mathbf{y}^T/\partial \mathbf{P})_{i,j} \triangleq [\partial y_j/\partial p_{s_i}]$. By Eq. 11 we have

$$\left.\begin{aligned} &\frac{\partial \mathbf{y}^T}{\partial \mathbf{P}} = \mathbf{S}^T \boldsymbol{\phi} \mathbf{W}_1 \\ &\text{and} \quad \frac{\partial \dot{\mathbf{y}}^T}{\partial \mathbf{P}} = \mathbf{S}^T \boldsymbol{\phi} \mathbf{W}_2 \end{aligned}\right\} \quad \ldots \ldots (14)$$

in which $\mathbf{W}_1 \triangleq \int_{t_o}^{t_o+t_w} \mathbf{V}_1(t-\tau)\,d\tau$ and $\mathbf{W}_2 \triangleq \dot{\mathbf{W}}_1$.

Using Eqs. 11 and 14 in Eq. 13 gives

$$\mathbf{P} = -\mathbf{B}^{-1}[\mathbf{S}^T\phi(I_1 + I_2)\phi^T\mathbf{M}\mathbf{x}(t_o) + \mathbf{S}^T\phi(I_1' + I_2')\phi^T\mathbf{M}\dot{\mathbf{x}}(t_o) + \mathbf{S}^T\phi(I_d)] \quad (15)$$

in which $\mathbf{B} = \mathbf{S}^T\phi(I_1'' + I_2'')\phi^T S$;

$$\left.\begin{aligned} I_j'' &= \int_{t_o}^{t_o+T_p} \mathbf{W}_j(t)\,\mathbf{A}_j\mathbf{W}_j(t)\,dt, \quad j = 1, 2; \\ I_j' &= \int_{t_o}^{t_o+T_p} \mathbf{W}_j(t)\,\mathbf{A}_j\mathbf{V}_j(\tilde{t})\,dt, \quad j = 1, 2; \\ I_j &= \int_{t_o}^{t_o+T_p} \mathbf{W}_j(t)\,\mathbf{A}_j\mathbf{U}_j(\tilde{t})\,dt, \quad j = 1, 2; \\ \text{and} \quad I_d &= \int_{t_o}^{t_o+T_p} [\mathbf{W}_1(t)\,\mathbf{A}_1\mathbf{y}_d(t) + \mathbf{W}_2(t)\,\mathbf{A}_2\dot{\mathbf{y}}_d(t)]\,dt \end{aligned}\right\} \quad (16)$$

We observe that the first three integrals in (16) can all be evaluated for a given dynamic system and T_p in closed form. For a time invariant system with $\mathbf{f}_d = 0$, the on-line computations required to determine $\mathbf{P}$ then become very modest (in comparison with the computations required by feedback theoretic methods), because the coefficient matrices of $\mathbf{x}(t_o)$ and $\dot{\mathbf{x}}(t_o)$ in Eq. 15 can now be calculated off-line and stored. The determination of the control pulse vector, $\mathbf{P}$, at time, t_o, then requires the relatively simple on-line job of finding two matrix products and performing a matrix addition.

If the impulse, $\mathbf{P}t_w$, applied at time t_o, can be approximated as $\mathbf{P}'\delta(t - t_o)$, then $\mathbf{W}_1(t) = \mathbf{V}_1(\tilde{t})$, and $I_j' = I_j''$, $j = 1, 2$. The vector $\mathbf{P}'$ then can be evaluated using Eq. 15, and the pulse magnitude, $\mathbf{P}$, can be approximated as

$$\mathbf{P} \simeq \frac{1}{t_w}\mathbf{P}' \quad (17)$$

It is observed from Eq. 15 that the applied pulse control vector, $\mathbf{P}$, is proportional to the mass matrix, $\mathbf{M}$, to the displacement vector $\mathbf{x}(t_o)$, and to the velocity vector $\dot{\mathbf{x}}(t_o)$. The term related to the displacement vector then may be throught of as providing an additional stiffness to the system, while that related to the velocity vector may be thought of as providing a viscous damping-like force. The relative contributions of these two types of effects will depend on the weighting matrices, $\mathbf{Q}_1$ and $\mathbf{Q}_2$, used in the expression defining the cost function, J.

Weighting Matrices $\mathbf{Q}_1$ and $\mathbf{Q}_2$.—Different weighting matrices, $\mathbf{Q}_1$ and $\mathbf{Q}_2$, yield different cost functions which can be used for computing the pulse vector, $\mathbf{P}'''$. For instance, $\mathbf{Q}_2 = 0$ would lead to the minimization of the weighted rms displacements. In a noisy measurement environment, $\mathbf{Q}_1$ then can be taken as the inverse of the covariance matrix of the measurement noise. The integrals, I_j, I_j', and I_j'', $I_j'' = 1, 2$, for the determination of $\mathbf{P}'$, when $\mathbf{Q}_1 = \mathbf{M}$, are given in Appendix I.

A particularly simple relation for determining $\mathbf{P}'$ (Eq. 17) results when $\mathbf{Q}_1 = \mathbf{K}$ and $\mathbf{Q}_2 = \mathbf{M}$ (assuming that $\mathbf{K}$ and $\mathbf{M}$ are symmetric, positive, and definite), so that J represents the vibrational energy of the system. The matrices $\mathbf{A}_1$ and $\mathbf{A}_2$ then equal Λ and I, respectively. The corresponding integrals, I_j, I_j', and I_j'', $j = 1, 2$, are evaluated in closed form Appendix II. One can observe that for this case

$$I_1 + I_2 = 2Z_1\Lambda \int_0^{T_p} \mathbf{V}_1^2 \, dt \quad \text{(18)}$$

Comparing this with the expression for $I_1' + I_2'$, for lightly damped systems

TABLE 1.—Characteristics of System

i (1)	M_i (2)	K_i (3)	C_i (4)	T_i, in seconds (5)	ξ_i, as a percentage (6)
1	1	50	1.0	1.9	3.28
2	1	75	1.5	0.757	8.29
3	1	100	2.0	0.487	12.89
4	1	100	2.0	0.360	17.44

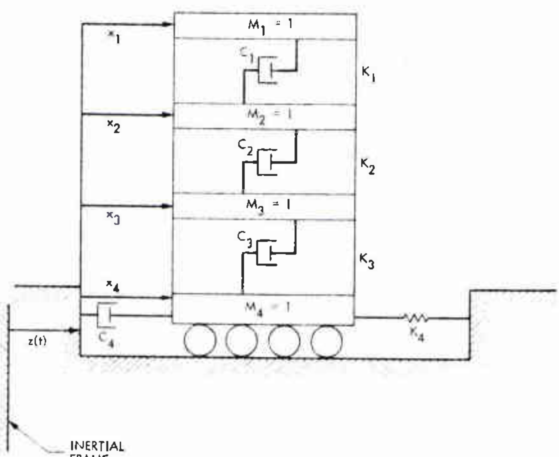

FIG. 1.—Four Degree-of-Freedom Model of Mechanical Subcomponent

($\xi_i << 1$) vibrating primarily in their lower modes, the contribution to $\mathbf{P}'''$ from the first term on the right hand side of Eq. 15 involving $\mathbf{x}(t_o)$, in general would be small compared with the contribution from the second term involving $\dot{\mathbf{x}}(t_o)$. The minimization of the energy of the system (when $\mathbf{f}_d = 0$) thus requires that the pulse magnitude essentially be proportional to the mass matrix and the velocity vector $\dot{\mathbf{x}}(t_o)$. This velocity-proportional force exhibits the damping-like nature of the pulse control.

The case when $\mathbf{Q}_1 = \mathbf{K}$ and $\mathbf{Q}_2 = 0$ defines J as the potential energy of the system. In that case, for determining $\mathbf{P}'$, I_2, I_2' and I_2'' are identically zero and I_1, I_1' and I_1'' are the same as those in Appendix II.

Chatter Suppression.—The control achieved by this strategy using two different cost functions is illustrated for a mechanical system modeled as a four degree-of-freedom oscillator (Fig. 1). The characteristics of the system are provided in

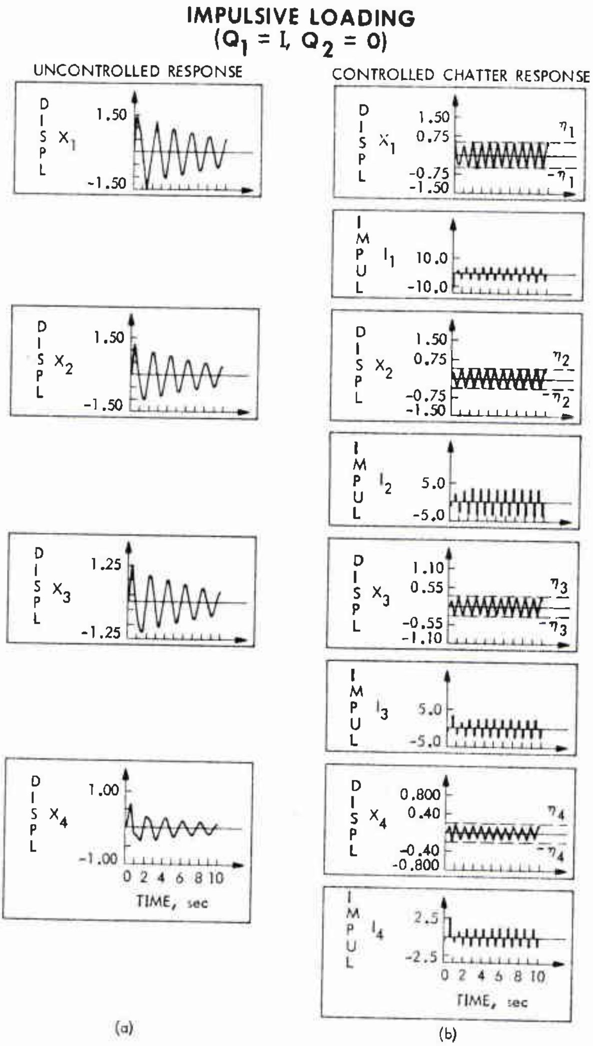

FIG. 2.—(*a*) Uncontrolled Response of System of Fig. 1 when Subjected to an Impulse of 10 Units Applied at Mass M_1; (*b*) Controlled Response and Control Time History of Impulses Showing Chatter when $Q_1 = I$, $Q_2 = 0$

Table 1. The system is subjected to an impulse at zero time of ten units applied to mass M_1. The parameters, t_w and T_p, are chosen to be 0.05 sec and 0.3 sec with $S = I_{N \times N}$. The constants, b_i, $i = 1, \ldots, N$, are each taken to be

zero, while the a_i's are each taken to be unity with $\eta_1 = 0.5$, $\eta_2 = 0.4$, $\eta_3 = 0.3$, and $\eta_4 = 0.2$ length units.

Weighting Matrices $Q_1 = I$ and $Q_2 = 0$.—Fig. 2 shows the controlled and uncontrolled responses of the system, together with the impulse control time histories ($I_i, i = 1, 2, 3, 4$), required at the various masses M_i, $i = 1, 2, 3, 4$.

It may be observed that while the pulse control quickly brings the amplitude levels of the responses of the various masses to their threshold amplitudes,

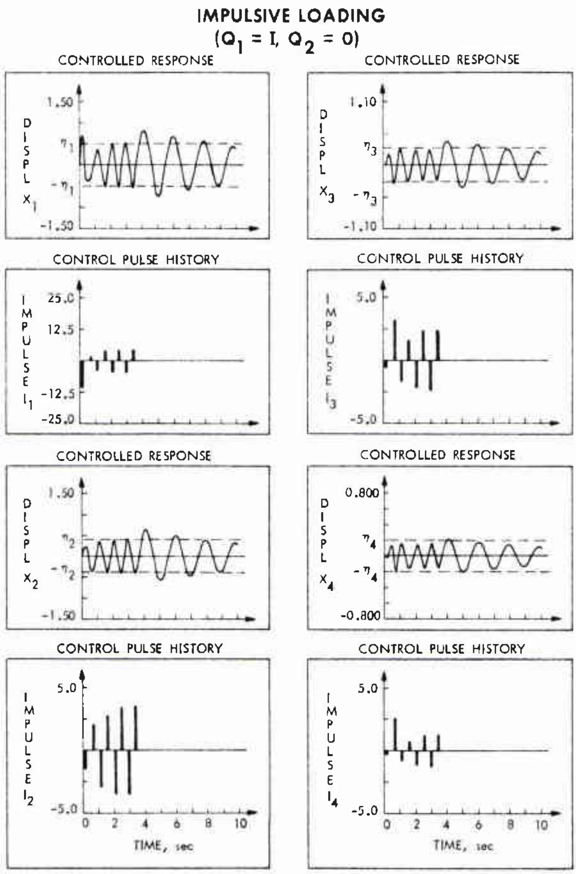

FIG. 3.—Chatter Suppression for System Subjected to 10 Unit Impulse at Mass M_1 with $Q_1 = I$, $Q_2 = 0$

the motion of the system is perpetuated by the alternate pulsing done by the control, despite the fact that the forcing functions, $f_d(t)$ and $f_s(t)$, are identically zero. The controlled response, though within the amplitude threshold values at the various mass points, will, in this case, exceed the uncontrolled response for large times, depending on the values of η_i and ξ_i. This "chatter phenomenon," which is common in bang-bang types of control systems, needs to be suppressed. Ideally, one would want to design the control system so that, for such an impulsive loading, after the preassigned threshold levels of response, η_i, are reached, the pulse control is cut off allowing the motions to die down by virtue of the damping present in the system.

One way of achieving this end is to track the system response, and when the system goes into a chatter mode of vibration, to set the pulse amplitude to zero. This sudden drop in the pulse amplitude will, in general, cause an overshoot in the system response (beyond η_i levels) to occur. If this overshoot

IMPULSIVE LOADING
(Q_1 = K, Q_2 = M)

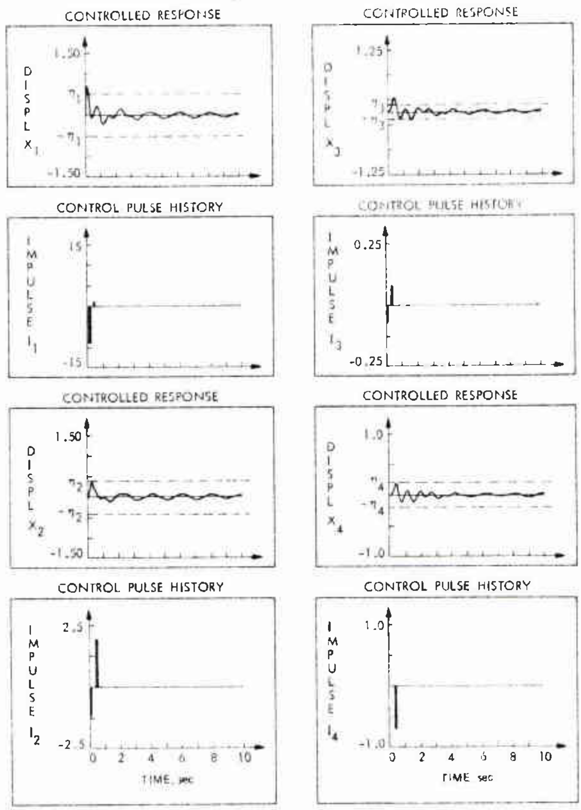

FIG. 4.—Controlled Response for Impulsive Loading Using Chatter Suppression with Q_1 = K, Q_2 = M

is less than α_i times $\eta_i(\alpha_i > 1$ is a preassigned constant), then the pulsing is stopped for a "dead period" of time, t_d. However, if a large overshoot (which conceivably could be caused by stochastic excitations occurring in the period, t_d) occurs at any node, the pulsing is continued in abeyance of the "dead time." The values of t_d and α_i to be chosen for the control logic will depend on the system characteristics, η_i, and the allowable overshoot in response beyond η_i, which is deemed nondamaging to the structure. In practice, the η_i can be chosen sufficiently small so that adequate control is achieved.

Fig. 3 shows the aforementioned implementation of the control logic with $t_d = 5.0$ sec., and $\alpha_i = 2.0$, $i = 1, 2, 3, 4$. At about 3.5 sec, the chatter mode is recognized and the pulse amplitudes are set to zero. This yields a slight response overshoot beyond η_i; from then on, the system is left to come to rest on its own without further control.

Weighting Matrices $Q_1 = M$ and $Q_2 = K$.—Using the same control parameters as before ($S = I_{N \times N}$), the pulse control vector **P** is obtained each time the response amplitudes, x_i, exceed η_i, by using Eq. 17. One can observe in Fig.

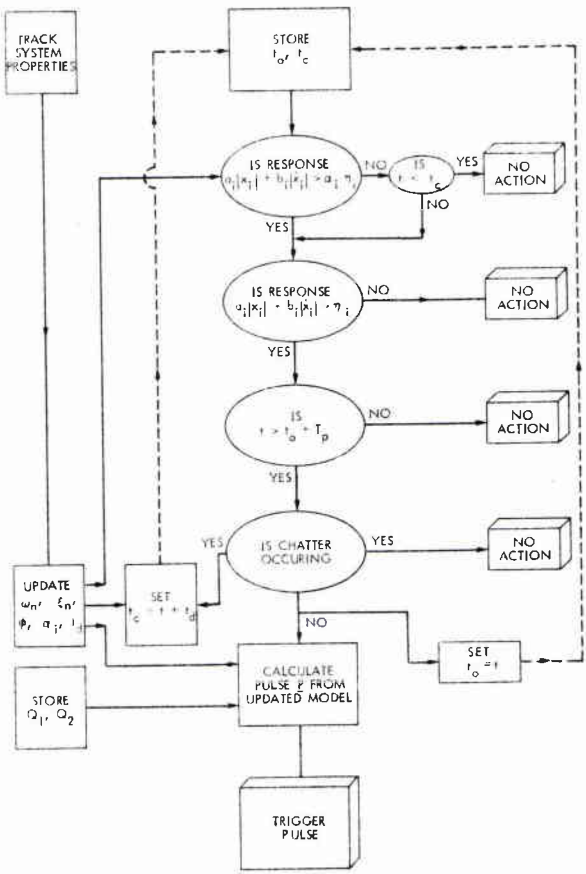

FIG. 5.—Flow Chart for Control Logic

4 that, unlike in the previous case, no significant chattering effect occurs. This is due to the fact that the pulse control acts primarily as an effective viscous damper, and does not in large measure rely on reducing the response amplitudes by increasing the effective stiffness of the system.

For time-variant systems, the system properties would need to be tracked, and the updated values of ω_n, ξ_n and the modal matrix would be used each time that the pulse magnitude $\mathbf{P}^m$ is required to be determined. As the system properties change, updated values of α_r and t_d would also be required. The flow chart for the control logic is illustrated in Fig. 5.

Reduced Order Modeling Effects

The closed form solution of the control pulse vector, **P** is computationally efficient to determine and one can, in theory, utilize the previously described technique to control large multidegree-of-freedom systems. However, when dealing with such systems, it is often difficult to obtain accurate information about the complete set of eigenvalues and eigenvectors which characterize the system dynamics. Most continuous systems, when discretized to Eq. 1, lead to reduced order models (ROM). Furthermore, for band limited inputs, it may suffice to use a smaller number of modes, K, ($K < N$), for an adequate description of the system response, **x**.

Thus, in most situations, lower order models will be utilized, where ϕ now becomes an $N \times K$ matrix containing the K modes that significantly contribute to the structural response. As the pulse control would then be calculated on the basis of the ROM, it is necessary to ensure that the pulse, **P**, so computed, when actually applied to the physical system, does not have an adverse effect on the unmodelled modes of the system, does not cause these modes to perhaps accumulate excessively large amounts of energy nor even perhaps cause the response of such modes to become unstable.

To broach this problem, let us define a continuous structural system through the one-dimensional equation (with suitable boundary conditions) as

$$L[u] + D[\dot{u}] = \rho(x)\frac{\partial^2 u}{\partial t^2} - s(x,t) - p(x,t) \quad \text{(19)}$$

in which x is the spatial coordinate, $p(x)$ = the mass density, and $u(x,t)$ = the displacement response created by the application of the load $s(x,t)$ and the control force $p(x,t)$. Assuming that the system has normal classical modes

$$u(x,t) = \sum_{1}^{\infty} A_n(t)\cdot\Phi_n(x) \quad \text{(20)}$$

in which $\langle\Phi_m, L[\Phi_n]\rangle = -\omega_n^2\,\delta_{mn}$; and $\langle\Phi_m, D[\Phi_n]\rangle = -2\omega_n\xi_n\delta_{mn}$, with Φ_n normalized so that $\langle\Phi_n, \rho\,\Phi_m\rangle = \delta_{mn}$. The modal coordinate, $A_n(t)$, then satisfies the equation

$$\ddot{A}_n(t) + 2\omega_n\xi_n\dot{A}_n(t) + \omega_n^2 A_n = \langle\Phi_n, s(x,t)\rangle$$

$$+ \langle\Phi_n, p(x,t)\rangle, \quad n = 1, 2, \ldots\infty \quad \text{(21)}$$

Assuming that the first K modes contribute significantly to the response caused by the input, $s(x,t)$, let us say that these K modes are used for the ROM and, therefore, for the calculation of the control force $p(x,t)$. Then, clearly, the inner products $\langle\Phi_n, s(x,t)\rangle$, $n > K$, are small, for if they were large we would opt to control the motion of those modes. Noting that the control force, $p(x,t)$, comprises pulses of duration t_w, the energy input, e_n, to the nth mode, caused by a pulse, $p(x,t)$, which is applied at time t_o, can be approximated by

$$e_n(t_o) = \frac{1}{2}\,[i_n(t_o)]^2 \triangleq \frac{1}{2}\,[\langle \Phi_n, p^m(x, t_o)\rangle\, t_w]^2 \quad \ldots\ldots\ldots\ldots\ldots \quad (22)$$

in which $p^m(x, t_o)$ = the magnitude of the pulse control (of duration t_w) applied at location x at time t_o. The minimum pulse spacing being $T_p \triangleq 2\pi/\omega_p$, the

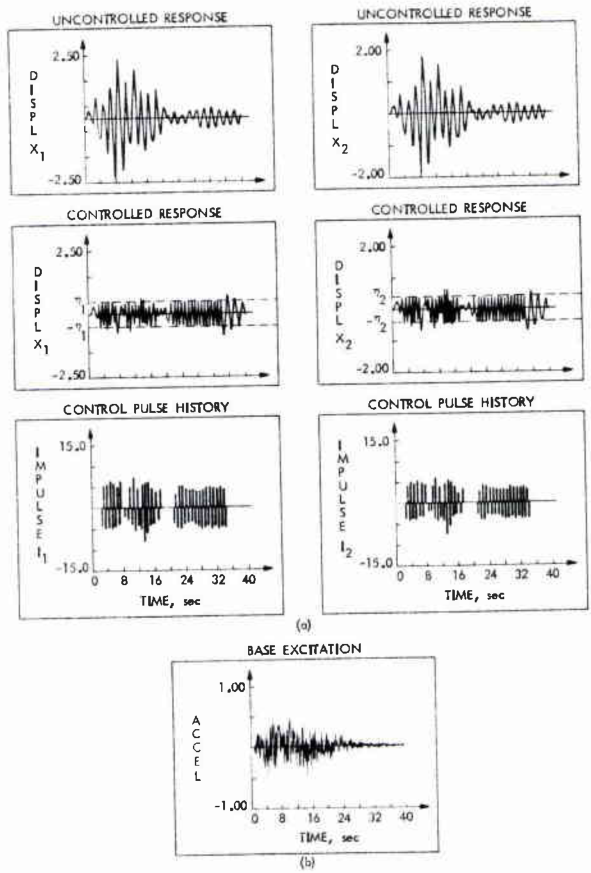

FIG. 6.—Uncontrolled Responses, Controlled Responses, and Control Pulse History at Masses M_1 and M_2 when System of Fig. 1 is Subjected to Stochastic Base Acceleration Shown ($Q_1 = I = M$, $Q_2 = 0$)

equation governing the growth of energy, E_n, in the nth mode can be expressed as

$$\frac{dE_n}{dt} \simeq -2\omega_n \xi_n E_n + \frac{\omega_p}{2\pi}\, e_n(t) \quad \ldots\ldots\ldots\ldots\ldots \quad (23)$$

As there is an upper bound on the magnitude of the pulse $p^m(x, t_o)$, that can be produced, i_n is bounded above by i_{max}. Eqs. 22 and 23 yield

$$E_n(t) \leq \frac{1}{2} i_{max}^2 \cdot \frac{1}{4\pi\xi_n} \left(\frac{\omega_p}{\omega_n}\right) [1 - \exp(-2\omega_n \xi_n t)] \qquad (24)$$

For large times ($t \rightarrow \infty$) the energy in the nth mode is not only bounded, but falls off inversely, as the modal frequency. Thus, even if the pulse control

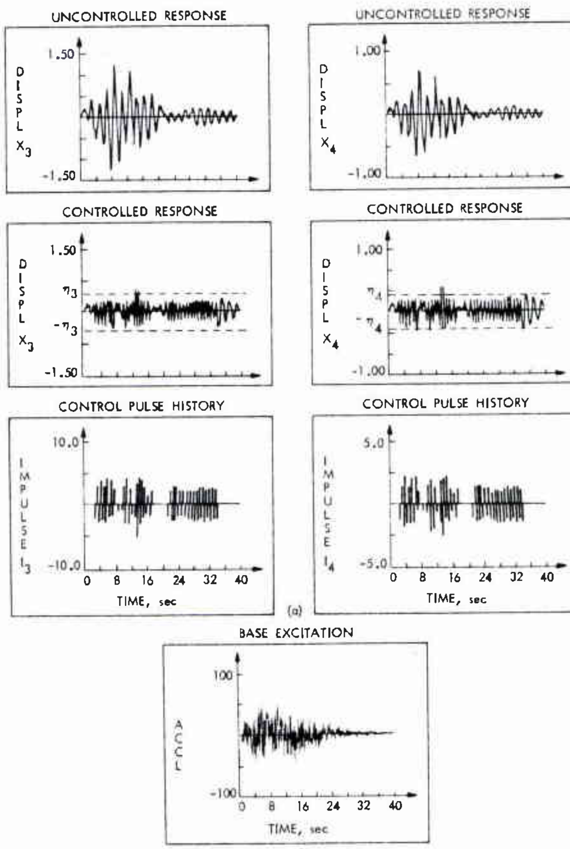

FIG. 7.—Uncontrolled Responses, Controlled Responses, and Control Pulse History at Masses M_3 and M_4 when System is Subjected to Stochastic Base Acceleration Shown ($Q_1 = I = M$, $Q_2 = 0$)

is computed on the basis on an ROM, the pulses would not cause the energies in the higher modes to become unbounded for a damped oscillating system.

Application to Structural System

The structural system represented by Table 1 is subjected to a base acceleration, $\ddot{z}(t)$, comprised of a time modulated Gaussian white noise signal, as shown in Fig. 6(*b*). This same base excitation will be used for all the examples in this sequel. The control parameters are set to; $t_w = 0.05$ sec, $T_p = 0.3$ sec, $a_i = 1, \forall i$; $b_i = 0, \forall i$; and $\eta_1 = 0.5$, $\eta_2 = 0.4$, $\eta_3 = 0.3$, and $\eta_4 = 0.2$ length units.

Figs. 6 and 7 show the uncontrolled and controlled responses, x_i, $i = 1, 2, 3, 4$, for the case in which the actuators are located at each of the masses ($S = I_{N \times N}$) with $\mathbf{Q}_1 = I\ (= \mathbf{M})$, and $\mathbf{Q}_2 = 0$.

The time histories of the impulses ($I_i, i = 1, 2, 3, 4$), required to be applied

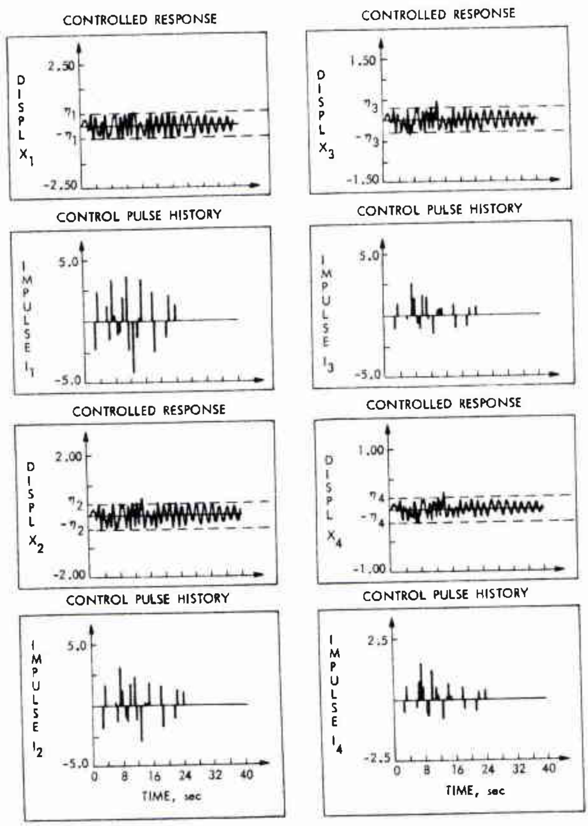

FIG. 8.—Controlled Response of System when Subjected to Base Acceleration of Fig. 6(*b*), with $\mathbf{Q}_1 = \mathbf{K}$ and $\mathbf{Q}_2 = \mathbf{M}$, Using Actuators at each Mass

to control the system, are also shown. Comparing the uncontrolled and controlled responses, we find that whereas the pulse control has effectively curtailed the amplitudes of motion to lie within the threshold values chosen (η_i), the controlled responses continue to persist at these threshold values for considerable lengths of time (between $t \sim 20$ sec and $t \approx 36$ sec) before the chatter mode is suppressed and the system eventually ($t \approx 36$ sec) allowed to come to rest.

The controlled response to the same base excitation of Fig. 6(*b*), using the same control parameters ($S = I_{N \times N}$) as before but now minimizing the energy of the system by setting the weighting matrices to $\mathbf{Q}_1 = \mathbf{K}$ and $\mathbf{Q}_2 = \mathbf{M}$, is indicated in Fig. 8. We observe that the control pulses required are fewer in number and the control on the whole is better than that achieved by minimization of the rms displacements.

Fig. 9, while using the same J as previously mentioned, shows the controlled responses when only two actuators are used, one at mass $\mathbf{M}_1$ and the other

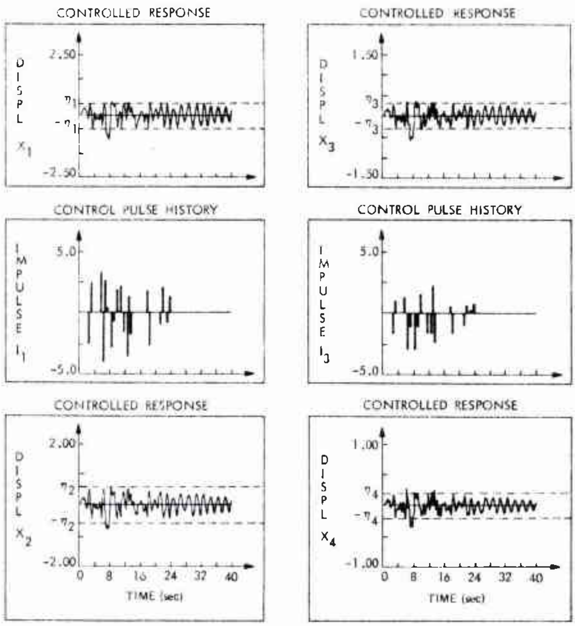

FIG. 9.—Controlled Response of System when Subjected to Base Acceleration of Fig. 6(*b*), with $\mathbf{Q}_1 = \mathbf{K}$, $\mathbf{Q}_2 = \mathbf{M}$, Using Actuators at Masses $\mathbf{M}_1$ and $\mathbf{M}_3$ Only

at mass $\mathbf{M}_3$. The selection matrix, **S**, is now a 4 × 2 matrix. The control pulse time history is also indicated.

Review and Conclusions

A simple preliminary adaptive open-loop pulse control method for limiting the response of an N degree-of-freedom system (with classical normal modes) by using M actuators at preassigned locations, is investigated. The control algorithm, while being heuristic in certain respects, yields the pulse vector, **P**, in closed form, thereby significantly reducing the on-line computational job when compared with optimal control theoretical methods. The computational advantage arises from the fact that a major part of computations can be performed off-line, and the necessary matrices stored. Then, the only significant on-line computations needed comprise the multiplication of these stored matrices with the observed displacement and velocity vectors. The technique thus also yields considerable savings in on-line core storage requirements.

While the solution for **P** has been carried out for a general form of the cost function, J, specific attention is devoted to the determination of **P**: (1) To minimize the rms system response in the interpulse interval; and (2) to minimize the system energy in the interpulse interval. It is found that the first criterion would generally require the suppression of chatter that may be induced by the pulsing technique. The second cost function creates a pulsing force which primarily generates an effective damping for lightly damped systems ($\xi_i << 1$), the force being proportional to the velocity vector at the time of pulsing. The tendency for chatter is much reduced in this case.

Whereas no consideration has been given to the location of the actuators, it is clear that a particular mode of vibration can only be controlled if the pulse control vector has a nonzero component along that mode [5]. The problem of finding optimal actuator locations will be left for a future communication. Similarly, other triggering criteria such as a triggering threshold related to the relative internodal displacements, suggest themselves for different applications. These too will be reported on later.

The main advantages of the method are the following:

1. The computational requirements are very modest, making the method suitable for large multidegree of freedom systems.
2. Both deterministic and stochastic excitations can be handled with equal ease.
3. The control technique does not create an adverse effect on the unmodeled modes of the system.
4. The method can be extended to time varying systems and nonlinear systems if equivalent time invariant linear characterizations can be obtained. These updated equivalent system properties, if tracked, would be used for the calculation of the pulse control vector.
5. The technique is simple to implement, perhaps leading to higher system reliability.

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Appendix I.—Integral Evaluation

Taking $\mathbf{W}_1(t) = \mathbf{V}_1(\tilde{t})$, $\mathbf{A}_1 = \phi^T \mathbf{M} \phi = I$, and $\mathbf{A}_2 = 0$ we have

$$I_1 = \int_{t=t_o}^{t=t_o+T_p} \mathbf{V}_1(\tilde{t})\,\mathbf{A}_1\,\mathbf{U}_1(\tilde{t})\,dt = \int_0^{T_p} \mathbf{V}_1(t)\,\mathbf{U}(t)\,dt + \mathbf{Z}_1 \int_0^{T_p} \mathbf{V}_1^2(t)\,dt \quad (25a)$$

$$I_1' = \int_0^{T_p} \mathbf{V}_1^2(t)\,dt \quad (25b)$$

$$I_1'' = I_1', \quad (25c)$$

$$\text{and} \quad I_2 = I_2' = I_2'' = [0] \quad (25d)$$

in which $\int_0^{T_p} \mathbf{V}_1(t)\,\mathbf{U}(t)\,dt = \lceil a_n \rfloor$; and $\int_0^{T_p} \mathbf{V}_1^2(t)\,dt = \lceil b'_n \rfloor$ (26)

Denoting $S_1 = -2\omega_n \xi_n$, $S_2 = 2\omega_{d,n}$, and $D = S_1^2 + S_2^2$, we get

$$a_n = \frac{1}{2} \exp \frac{(S_1 T_p)[(S_1 \sin S_2 T_p - S_2 \cos S_2 T_p) + S_2]}{(\omega_{d,n} D)} \quad (27a)$$

$$b_n = \frac{1}{4\omega_n \xi_n \omega_{d,n}^2} [1 - \exp(S_1 T_p)] \quad (27b)$$

$$c_n = \frac{\frac{1}{2} \{S_1 - \exp(S_1 T_p)[S_1 \cos S_2 T_p + S_2 \sin S_2 T_p]\}}{(\omega_{d,n}^2 D)} \quad (27c)$$

and $b'_n = b_n + c_n$ (27d)

Appendix II

Taking $\mathbf{W}_1(t) = \mathbf{V}_1(\tilde{t})$, $\mathbf{Q}_1 = \mathbf{K}$, and $\mathbf{Q}_2 = \mathbf{M}$ for the determination of $\mathbf{P}'$, we have

$$I_1 = \Lambda \int_0^{T_p} \mathbf{V}_1(t)\,\mathbf{U}(t) + \Lambda Z_1 \int_0^{T_p} \mathbf{V}_1^2(t)\,dt;$$

$$I_2 = \Lambda Z_1 \int_0^{T_p} \mathbf{V}_1^2(t)\,dt - \Lambda \int_0^{T_p} \mathbf{V}_1(t)\,\mathbf{U}(t)\,dt \quad (28a)$$

$$I'_1 = I''_1 = \Lambda \int_0^{T_p} \mathbf{V}_1^2(t)\,dt \quad (28b)$$

and $I'_2 = I''_2 = \int_0^{T_p} \mathbf{U}^2(t)\,dt - 2Z_1 \int_0^{T_p} \mathbf{V}_1(t)\,\mathbf{U}(t) + Z_1^2 \int_0^{T_p} \mathbf{V}_1^2(t)\,dt$ (28c)

All integrals in the above relations are defined in Appendix I except

$$\int_0^{T_p} \mathbf{U}^2(t)\,dt = \lceil d_n \rfloor$$

in which $d_n = (b_n - c_n)\,\omega_{d,n}^2$ in the notation of Appendix I.

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Appendix IV.—Notation

The following symbols are used in this paper:

a_i, b_i = preassigned constants;
$\mathbf{C}$ = N × N damping Matrix;
E_n = energy in the nth mode response;
e_n = energy input to the nth mode;
$\mathbf{f}_d(t)$ = N × 1 deterministic vector component of dynamic load;
$\mathbf{f}_s(t)$ = N × 1 stochastic vector component of dynamic load;
J = Minimization Functional;
$\mathbf{K}$ = N × N stiffness matrix;
$\mathbf{M}$ = N × N mass matrix;
$p^m_{s_i}$ = pulse magnitude which is applied at node s_i;
$p^L_{s_i}, p^U_{s_i}$ = lower and upper bounds on pulse magnitudes to be applied at node s_i;
$\mathbf{Q}_1, \mathbf{Q}_2$ = symmetric, positive definite weighting matrices;
$\mathbf{r}$ = N × 1 Vector of control forces;
$\mathbf{S}$ = N × M selection matrix;
T_p = minimum interpulse time;
T_N = undamped natural period of vibration of the nth mode;
t_d = dead time for chatter suppression;
$\mathbf{x}$ = N × 1 displacement vector;
$\mathbf{y}$ = modal coordinate vector;
α, β = preassigned constants;
ξ_i = percentage of critical damping in mode i;
$\omega_n, \omega_{d,n}$ = undamped and damped natural frequencies of vibration; corresponding to the nth mode; and
$\delta(t)$ = Dirac Delta Function.