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Incorporating Managerial Thinking in Prediction and Control: Case Study of Market Penetration

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Abstract. Managerial strategies, especially at the higher echelons of management, are often linguistically stated. This is because they need to be based on information which often defies quantification. Such verbal strategies and qualitative information have often been found to be difficult to incorporate in quantitative models. Thus, the quantitative effects of implementing one strategy as opposed to another have generally been difficult to forecast.

In this paper, we show that, through the use of fuzzy logic, we can incorporate such qualitative (linguistically stated) information. Furthermore, we show that a fuzzy controller can be designed so as to reach desired goals while being cognizant of linguistically stated strategies, scenarios, and decision rules as well as quantitative data types.

The approach is applied to the modeling and control of market penetration, a field which has attracted considerable attention in recent years.

Key Words. Managerial strategies, qualitative information, quantitative predictions, fuzzy controller, market penetration, nonuniform influence parameter.

1. Introduction

Managerial strategies and decisions are usually based on quantitative data, like market share and ROI, and on qualitative data such as global market conditions, corporate belief systems, etc. Often such strategies, especially at the higher echelons of management, are stated linguistically to

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take into account different perspectives such as corporate risk taking attitudes, current practice, operating norms, and corporate culture—perspectives which involve data types that often defy precise quantification. Yet, from a survival standpoint, it may arguably be the linguistically stated strategies and data types which have the most long-range impact on a corporation.

Because such data, and the consequent decisions (strategies) which they engender, lack quantitative structure, it is often difficult to incorporate them in quantitative models and assess the impact of different linguistically stated strategies on tangible outcomes such as market performance. In this paper, we show that both the qualitatively stated information, strategies, and scenarios, and the quantitative information, such as market share, can be simultaneously used to provide quantitative model-based predictions. We do this through the use of fuzzy logic and show that such an approach can be used to control market outcomes along desired directions.

Specifically, we consider a two-product world and show how the market penetration of a product can be controlled through the use of a fuzzy controller which incorporates both linguistic and quantitative information. Reaching desired marketing goals through marketing activities based on linguistically stated strategies and information is shown to be indeed possible. This allows us to compare the efficiency of different (linguistically stated) strategies by giving a prediction of the market performance that each generates. Furthermore, identification of strategies under which the desired goals cannot be met is also shown to be possible.

The model of market penetration that we use to illustrate our ideas is a variant of a model prevalent in the field, principally due to Mahajan, Muller, and Bass (Refs. 1–9). In the next section, we present this model. Then, we show how on the basis of both quantitatively stated data and qualitative corporate strategies we can design a fuzzy controller to affect the market penetration of one of the products by controlling the nonuniform influence parameter. We demonstrate our approach and results through simulations. Comparisons of the effectiveness of the strategies used are provided.

2. Nonuniform Diffusion Model for Competing Products

For the last three decades many studies have attempted to use mathematical models to describe the diffusion process of an innovation and to forecast the market share the innovation can achieve; see Ref. 10 for an introduction to diffusion of innovations. One of the first models in the area of innovation diffusion is the Bass diffusion model (Ref. 1). It describes the

diffusion process of an innovation in terms of the market penetration using a differential equation. In several recent papers (Refs. 4, 6, 11), additional features were introduced into the Bass diffusion model to include more reasonable assumptions about a diffusion process and to get results which better fit actual diffusion processes. Also, other attempts (Refs. 2, 5, 7, 8, 12, 13) have been made to take into consideration the influence of marketing activities, several products, or intermarket influence in order to forecast the behavior of the diffusion process of a product.

The basic assumption of Bass (Ref. 1) is that the conditional probability of adoption of an innovation at time t is related to the fraction of potential users who have already adopted the innovation. He described the innovation diffusion process, which excludes repeated purchases, using the following differential equation:

$$dN(t)/dt = a(\bar{N} - N(t)) + (b/N)N(t)[\bar{N} - N(t)], \tag{1}$$

or

$$\begin{aligned} dP(t)/dt &= a[1 - P(t)] + bP(t)[1 - P(t)] \\ &= [a + bP(t)][1 - P(t)], \end{aligned} \tag{2}$$

where $N(t)$ is the cumulative number of adopters at time t, $\bar{N}$ is the number of potential adopters, a and b are the so-called coefficient of innovation and coefficient of imitation, respectively, and $P(t) = N(t)/N$ is the fraction of potential users who have adopted the product at time t; it is also called the penetration of the innovation.

The term $a[1 - P(t)]$ describes the adoption by innovators; the term $bP(t)[1 - P(t)]$ describes the adoption by imitators. However, the Bass model explains the innovation diffusion by means of communication channels; therefore, Mahajan, Muller, and Bass (Ref. 6) labeled a and b the coefficient of external influence and the coefficient of internal influence, respectively, which better fits the above characterization. The coefficient of external influence refers to the dissemination of information about the product via mass media, while the coefficient of internal influence captures the dissemination due to word-of-mouth interpersonal communication. Empirical observations indicate that b takes on only positive values; see, e.g., Refs. 4–5. Accordingly, it will be assumed that $b \geq 0$, where $b = 0$ means that there is no word-of-mouth influence at all.

Easingwood, Mahajan, and Muller (Ref. 4) proposed that the coefficient of internal influence be a function of penetration. They described the coefficient of internal influence as follows:

$$w(t) = b[N(t)/\bar{N}]^{\gamma} = bP(t)^{\gamma}, \tag{3}$$

where γ is a constant.

Substitution of Eq. (3) into Eq. (2) gives the nonuniform influence diffusion model (NUI)

$$dP(t)/dt=[a+bP(t)^{\gamma+1}][1-P(t)]=[a+bP(t)^{\alpha}][1-P(t)], \quad (4)$$

for $\alpha=\gamma+1\geq 0$.

The result of incorporating the time-varying coefficient of internal influence is that the NUI model allows the adoption rate curve to be symmetrical or nonsymmetrical and allows the maximum rate of penetration (i.e., the point with the highest sales volume) to occur at any time. The parameter α is called the nonuniform influence parameter. For $0<\alpha<1$, the maximum rate of change of the penetration occurs at an earlier stage of the process and the coefficient of internal influence is initially high and then decreasing; for $\alpha>1$, the maximum rate of change occurs at a later stage and is smaller, compared to the case $0<\alpha<1$, and the coefficient of internal influence increases at first and then decreases. For an illustration of this result, see, e.g., Easingwood, Mahajan, and Muller (Ref. 4).

So far, the NUI model describes the innovation diffusion process for one infrequently purchased product, and it appears to allow all shapes of diffusion processes. Mahajan and Peterson (Ref. 7) generalized the basic Bass model to the situation of two substitutive products, which yields a system of two differential equations for the diffusion processes of the two products. Mahajan and Muller (Ref. 6) used a similar approach to model the diffusion process for one product on two different markets. In both situations, there is an additional influence of the diffusion of the substitutive product on the diffusion of the product and of the diffusion in the second market on the diffusion in the first market, respectively.

In this paper, we use a generalization of their NUI model to the situation of two competing products by including an internal word-of-mouth effect of the products on each other,

$$dP_1(t)/dt=[a+bP_1(t)^{\alpha}-qP_2(t)^{\beta}][1-P_1(t)-P_2(t)], \quad (5)$$

$$dP_2(t)/dt=[p+qP_2(t)^{\beta}-bP_1(t)^{\alpha}][1-P_1(t)-P_2(t)], \quad (6)$$

where $P_1(t)$ and $P_2(t)$ denote the potential market shares for the two competing products with respect to the potential market size, a and p represent the constant coefficients of external influence, $bP_1(t)^{\alpha}$ and $qP_2(t)^{\beta}$ are the time-varying coefficients of internal influence (b and q are assumed to be nonnegative), and $\alpha\geq 0$, $\beta\geq 0$ are the constant nonuniform influence parameters.

The model (5)–(6) is so far independent of any activities taken by the company to promote its product. Horsky and Simon (Ref. 12) and Simon and Sebastian (Ref. 13) investigated the influence of advertising and other marketing activities on the diffusion process. In the empirical study by Horsky and Simon (Ref. 12), the effect of advertising was described as a means to influence the coefficient of external influence. The aspect of influencing the coefficient of internal influence was considered to be of secondary importance. However, Simon and Sebastian (Ref. 13) found in their study that advertising is more likely to influence the coefficient of internal influence in the intermediate life cycle of the innovation. Advertising changed the behavior of the diffusion process with a time lag of about three to six months.

Thus, the nonuniform influence parameters α and β could be thought of as being piecewise constant. Their values can be influenced through marketing activities; changes in the marketing activities result in changed values of α and β.

To summarize, the model (5)–(6) used in this paper is a nonuniform influence diffusion model for two competing products, in which the nonuniform influence parameters α and β are piecewise constant (i.e., constant only for a certain period), but can be changed by marketing decisions. We assume that each product is manufactured and marketed by an organization, the organizations each attempting to increase their market share.

In the next section, we develop a fuzzy control scheme which incorporates linguistically assessed scenarios, strategies, and beliefs so that, by controlling the parameter α appropriately (using fuzzy logic), a desired market penetration P_1 can be achieved.

3. Fuzzy Logic Controller

In the previous section, we presented an NUI model for market penetration with piecewise constant nonuniform influence parameters. Not yet answered is the question of how these piecewise constant parameters can be obtained to reach a given penetration objective, within the constrains of given managerial thinking.

3.1. Fuzzy Logic System. At every point in the diffusion process modeled by (5)–(6), we have information about the market shares of both products, and each company has a goal for the product market share it wants to achieve. To reach this goal, the company uses, among others, advertising and other marketing activities to promote its products.

With the goal and the actual market shares for both products in mind, the company has from experience an idea how the market will react to certain marketing activities and, therefore, what kind of marketing activities the company should employ. However, these beliefs often cannot be indexed numerically in an exact manner but are rather of the form:

> Our product has a medium market share P_1, the competitor has a higher market share P_2. Our goal is to further increase our market share; therefore, we should use more marketing activities to get a high influence on the market in order to reach our goal.

As seen in the above example, we are given the linguistic information in fuzzy terms. What is a *medium* or *high* market share? What does *high influence on the market* mean? How should the company orchestrate its strategy to increase the market share in a manner consistent with its linguistically stated goals?

We need to get, from numerical information about market share and desired market share and from fuzzy linguistic information, what actions the organization should take to achieve its marketing goals. The model (5)–(6) indicates that, by changing the numerical value for the nonuniform influence parameter through its marketing activities, the organization may be able to reach these goals. In other words, how do we change the numerical value of the nonuniform influence parameter with time, based on both the qualitative and quantitative information at hand?

One way to do this is to use the concept of fuzzy logic systems (FLS); see, for example, Refs. 14–16. The FLS enables the company to express its goals, beliefs, strategies, and perceived environment in linguistic terms. Linguistic information from the management can be used to create a base of fuzzy if-then rules (managerial beliefs) of the following form:

> If our market share P_1 is medium, if the market share P_2 of the competing product is high, and if the relative error $(P_1 - G)/P_1$ between market share P_1 and our goal G is negative but decreasing, then the influence of the marketing efforts on the market penetration is medium, where the connotation *medium*, *high* and *negative* may represent fuzzy sets.

The basic structure of a fuzzy logic system is shown in Fig. 1. The basic model uses only fuzzy sets; i.e., the input is given in terms of fuzzy sets, and the output is also a fuzzy set (Ref. 17).

Since we have numerical information for the market shares P_1 and P_2, relative error e, and change in the relative error de, and since we need a numerical ouput α, we have to modify our FLS to accommodate these numerical values. We have to add a fuzzifier to the input and a defuzzifier to the output. The fuzzifier translates numerical values into fuzzy sets; the defuzzifier translates a fuzzy set into a numerical output. Wang's description (Ref. 17) of a fuzzy logic system with fuzzifier and defuzzifier (FLS-FD) is shown in Fig. 2. A fuzzy logic system used as a controller is called a fuzzy logic controller, and we use the FLS-FD to control the diffusion process. Therefore, we shall refer to the FLS-FD as the fuzzy logic controller.

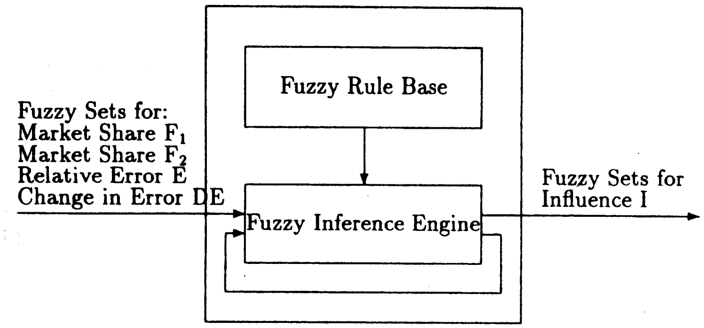

Fig. 1. Pure fuzzy logic system.

3.2. Design of the Fuzzy Logic Controller. We describe now the elements of our fuzzy logic controller, as depicted in Fig. 2. These elements are the fuzzy rule base, fuzzifier, fuzzy inference engine, and defuzzifier.

The fuzzy rule base consists of fuzzy if-then rules (managerial beliefs) of the previously described form. These rules contain the linguistic information supplied by management pertinent to matters such as corporate culture, operating procedures, attitudes to risk, etc.

The fuzzifier determines in which fuzzy regions the actual input values for the market shares of the two products, the relative error, and the change in the error lie.

The fuzzy inference engine takes all possible combinations of the previously determined fuzzy sets, compares them with the fuzzy rule base, and assigns to each combination the corresponding fuzzy region for the influence parameter α.

The defuzzifier finally uses all information about the input and output fuzzy sets and determines, using the information in terms of fuzzy sets, a

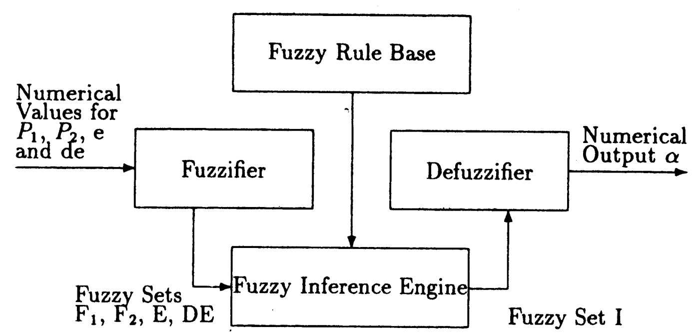

Fig. 2. Fuzzy logic system with fuzzifier and defuzzifier.

numerical output value. This is then the value for the nonuniform influence parameter α used to control the process described in Eqs. (5)–(6) to meet desired marketing goals.

Thus, the fuzzy logic controller enables us to determine the influence which the company should exercise, using the linguistic information about the market conditions and managerial beliefs supplied by the management and the actual numerical information about market shares of the two products.

Fuzzy Rule Base. The first step in constructing a fuzzy rule base is to divide the input and output spaces into fuzzy regions. We have then to assign a fuzzy membership function to each of the regions. The fuzzy membership function characterizes the fuzzy region and determines the degree of membership of an input in the fuzzy region. Thus, a fuzzy region is a generalization of an ordinary set; the membership function of a fuzzy region can take any value from the interval [0, 1], while the membership function of an ordinary set takes on only the two values 0 and 1.

After finding the fuzzy regions and the membership functions, the input–output relation has to be defined. These are fuzzy if-then rules (managerial beliefs) of the form

$$\text{If } P_1 \in F_1,\ P_2 \in F_2,\ e \in E,\ de \in DE, \text{ then } \alpha \in I, \tag{7}$$

where F_1, F_2, E, DE are fuzzy regions for the market shares, error, and change in the error, respectively, and I is the fuzzy region for the influence.

To generate the fuzzy rule base for the diffusion process, the management has to specify terms like *medium market share* or *high influence*, and it has to supply its reaction to certain situations in the form of if-then rules like (7).

Fuzzifier. The task of the fuzzifier is to convert each exact numerical information into a fuzzy set with an assigned membership value. For example, a market share P_1 results after the fuzzification in a fuzzy set F_1 with a membership value $\mu_{F_1}(P_1)$.

Our fuzzifier translates the numerical input (P_1, P_2, e, de) for the market shares P_1 and P_2, relative error e, and change in the error de into quadruples (F_1, F_2, E, DE) of fuzzy sets each with a corresponding quadruple of membership values $(\mu_{F_1}(P_1), \mu_{F_2}(P_2), \mu_E(e), \mu_{DE}(de))$. This means that the fuzzifier determines the fuzzy regions of (P_1, P_2, E, de) which are needed to use the fuzzy if-then rules (7) in the fuzzy rule base. We need to note that the input (P_1, P_2, e, de) corresponds in general to more than one quadruple of fuzzy sets (F_1, F_2, E, DE), since any of the input data can lie in one or more fuzzy regions.

Fuzzy Inference Engine. In order to construct the fuzzy inference engine, we have to define how a fuzzy implication is characterized.

For ordinary sets, we have the following definition of implication:

$$\text{If } x \in A \text{ and } A \subset B, \text{ then } x \in B.$$

This means that $A \Rightarrow B$; i.e., if we find an element in A, which has a membership value of 1 in this set, then this element is also an element of B with membership value 1.

In contrast to ordinary sets, there are several possible choices of how an implication for fuzzy sets can be defined. The characterizing element in these definitions is how the membership value of the input is used to find the membership value of the output in its fuzzy region.

The input for the fuzzy inference engine are the fuzzy sets (F_1, F_2, E, DE) with the corresponding membership values $(\mu_{F_1}(P_1), \mu_{F_2}(P_2), \mu_E(e), \mu_{DE}(de))$, computed by the fuzzifier. The fuzzy inference engine links then the fuzzy rule base with the fuzzy sets (F_1, F_2, E, DE) to determine the output fuzzy region I and assigns a membership value $\mu_{[(F_1,F_2,E,DE)\Rightarrow I]}(P_1, P_2, e, de)$ to the output fuzzy region, according to the used definition of fuzzy implication.

Defuzzifier. As we stated previously, the numerical input (P_1, P_2, e, de) can correspond to one or more quadruples of fuzzy sets (F_1, F_2, E, DE). For each of these quadruples, the fuzzy inference engine computes an output fuzzy region I with a corresponding membership value. We now need to convert this fuzzy region along with its membership value to a numerical output value α.

The defuzzifier now uses the output fuzzy regions I and the membership values to compute a numerical value α; i.e., the defuzzifier translates the output fuzzy sets to one numerical value.

The definitions for the four elements of the FLS-FD should be done in such a way that the system can describe real-life behavior for the diffusion process. The reader is referred to Wang (Ref. 17) for a more detailed description of the construction of the inference engine, fuzzifier, and defuzzifier.

All four elements together (rule base, fuzzy inference engine, fuzzifier, and defuzzifier) constitute our fuzzy logic controller, and the controller computes the nonuniform influence parameter α, based on the information about the market shares P_1 and P_2, error e, and change in the error de, and based on the supplied fuzzy rules.

3.3. Implementation of the Fuzzy Controller. We have seen in previous sections how to construct a fuzzy controller. In this section, we want to

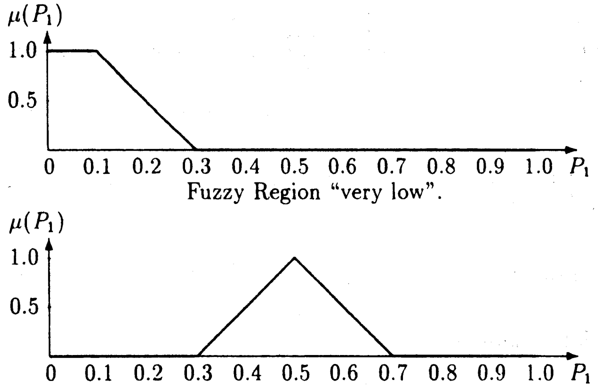

Fig. 3. Examples of two of the membership functions for the market share.

present the controller which is implemented for our simulation, i.e., how the four elements of the fuzzy controller are defined.

Two examples of the membership function for the market share are shown in Fig. 3. The membership function of the fuzzy region *very low* has a value of 1 for market shares between 0% and 10%. For a market share between 10% and 30%, the membership value decreases linearly to 0; market shares of more than 30% have a membership value of 0 in this fuzzy region defined by *very low*. The membership value of the market share in the fuzzy region *medium* is given by the triangular function depicted in Fig. 3. The membership functions of all five fuzzy regions of the market share are shown in Fig. 4.

Figures 4–7 exhibit the fuzzy regions and the membership functions for the market share, relative error, change in the error, and influence parameter α for the following simulations. We chose triangular membership functions only, although it is of course possible to use other membership functions, like functions of Gaussian or trapezoidal form, or to define different fuzzy regions. The membership functions and fuzzy regions in Figs. 4–6 are all

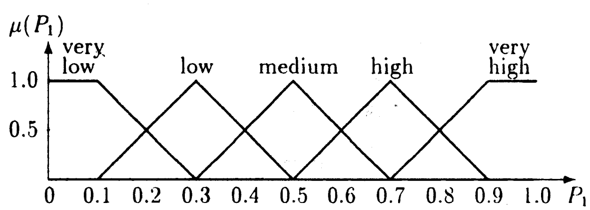

Fig. 4. Membership functions for the market share.

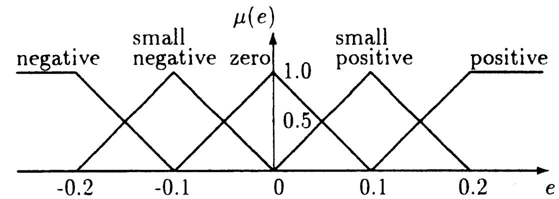

Fig. 5. Membership functions for the relative error.

symmetric, while the membership functions and fuzzy regions for the influence parameter α is nonsymmetric, as shown in Fig. 7. Our choice of the fuzzy regions and fuzzy membership functions was based on intuition and initial experimentation.

For example, in Fig. 4, a market share of 40% ($P_1 = 0.4$) has a membership value of 0.5 in the region *low market share*, a membership value of 0.5 in the region *medium market share*, and a membership value of 0 in all other regions.

The fuzzy inference engine is characterized by the interpretation of fuzzy implication. We chose the so-called mini-operation rule for fuzzy implication. The mini-operation rule is a good choice from an axiomatic point of view and is computationally simple (Ref. 17).

The number of fuzzy rules depends on the number of fuzzy regions defined for every input parameter. In our implementation (see Section 4.1), there are altogether 375 fuzzy rules for each strategy [$375 = 5 \times 5 \times 5 \times 3$, i.e., the product of the number of membership functions for both market shares, relative error, and change in the error (see Figs. 4–6)], which can be reduced to 285 (see Ref. 19), since some combinations of the market shares are not allowed due to the fact that the sum of the market shares has to satisfy $P_1 + P_2 \leq 1$. These rules may be tailored to reflect the organizational culture, market environment, and accepted managerial judgment. Risk taking attitudes are described in terms of conservative and aggressive strategies; see Section 4.1.

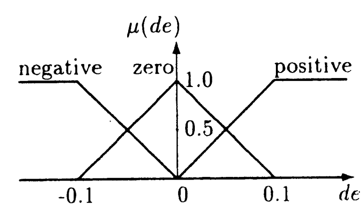

Fig. 6. Membership functions for the change in the error.

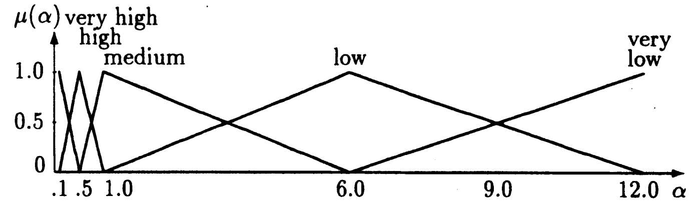

Fig. 7. Membership functions for the influence.

An example of the if-then fuzzy rules (managerial beliefs) which we have used for different strategies described in the next section is given below; see Ref. 19:

> If our market share P_1 is *medium*, if the market share P_2 of the competing product is *high*, and if the relative error between our market share P_1 and our goal G is *small* and *negative*, but *decreasing*, then the influence of our marketing efforts on the market is *medium*, if we pursue an aggressive strategy, and *low*, if we pursue a conservative strategy.

We have used a singleton fuzzifier which produces a simple fuzzy logic system; for $(P_1, P_2, e, de) \in (F_1, F_2, E, DE)$, the membership value of (P_1', P_2', e', de') in (F_1, F_2, E, DE) is defined to equal 1 if

$$(P_1', P_2', e', de') = (P_1, P_2, e, de)$$

and to equal 0 if

$$(P_1', P_2', e', de') \neq (P_1, P_2, e, de).$$

The implemented defuzzifier is the center average defuzzifier, which showed good performance in practical experiments and produces an easy and efficient fuzzy logic system (Ref. 17).

The above described fuzzy logic system with singleton fuzzifier, mini-inference rule, and center average defuzzifier is then given by

$$f(P_1, P_2, e, de) = \frac{\sum_{l=1}^{M} \bar{a}^l \min\{\mu_{F_1^l}(P_1), \mu_{F_2^l}(P_2), \mu_{E^l}(e), \mu_{DE^l}(de)\}}{\sum_{l=1}^{M} \min\{\mu_{F_1^l}(P_1), \mu_{F_2^l}(P_2), \mu_{E^l}(e), \mu_{DE^l}(de)\}}, \tag{8}$$

where $\mu_{Z^l}(z)$ denotes the membership value of z in the fuzzy region Z^l, $\bar{a}^l$ is the center of the fuzzy region I^l [i.e., the minimum value at which $\mu_{I^l}(\alpha^l) = 1$], and M is the number of all possible combinations of fuzzy regions (F_1, F_2, E, DE) in which P_1, P_2, e, de lie.

To summarize the implemented fuzzy logic system, the controller uses the numerical input values for the two market shares, relative error, and

change in the error, and the linguistic information about the market conditions as well as fuzzy managerial thinking to compute the nonuniform influence parameter by $\alpha = f(P_1, P_2, e, de)$, where f is defined as in (8).

4. Numerical Simulations

In the previous sections, we constructed a fuzzy controller to determine the values of the piecewise-constant nonuniform influence parameter α in (5) and (6) for the presented NUI model. In this section, we describe the general setup of the simulations and present the results for some selected diffusion processes.

The purpose of the simulations is to illustrate the usefulness of our approach in a variety of situations.

All simulations were performed on a Sun SPARCstation 10, using MATLAB. The integrator used for the system of differential equations (5)–(6) was MATLAB's efficient ODE45, adapted to include the parameters $\alpha, \beta, a, b, p, q$. ODE45 is based on the adaptive Runge–Kutta–Fehlherg scheme; see, for example, Forsythe, Malcolm, and Moler (Ref. 18) for a description of the algorithm. The program needed about 10 CPU seconds for a successful simulation. The program codes are listed in Ref. 19.

4.1. Strategies, Scenarios, Quantitative, and Qualitative Data. The qualitative data are the linguistically stated managerial beliefs and strategies, i.e., the linguistic information about the market conditions as well as fuzzy managerial thinking. Comparisons between two different linguistically stated strategies and two different perceived market scenarios have been conducted.

The two strategies applied are labeled *conservative* and *aggressive*. For each of them, a separate set of if-then rules that state management thinking has been created. The total set of 375 managerial beliefs incorporated in the simulations relative to each of the two strategies can be found in Ref. 19.

Using the aggressive policy, the company tries to reach its goal as quickly as possible at any cost. The company takes on the marketing activities it considers to be appropriate in the current situation, without taking into consideration the market share of its product. This means that the influence is not restricted by the market share of the product; i.e., the budget for the marketing activities for the product is independent of the revenues generated by the product. It should be remembered that higher influence (lower α value) eventually means more marketing activities, and therefore higher marketing costs.

If the company employs the conservative strategy, there is a correlation between the marketing budget and the market share. The company restricts the marketing budget to a certain percentage of the revenues generated by the product and spends only as much for marketing activities as necessary, even though it may take longer to reach the company goal, or in fact the goal may never be reached. That means that the coefficient of influence α is restricted to a certain subinterval of its domain.

The two scenarios considered are *competition* and *antimonopoly*. For the former, the error e is defined as the relative error $(P_1 - G)/P_1$ between market share P_1 and goal G. For the latter, it is defined as the relative error $(P_2 - G)/P_2$ between market share P_2 and goal G.

In the competition scenario, the goal of the company is to reach a certain market share for its product without any considerations of the market share of the second product. If the competing product is driven out of the market, the market form becomes a monopoly.

The antimonopolistic scenario avoids the monopolistic market by aiming to get the competing product market share to a certain margin, but without driving the competition out of the market. The goal for the company product is then to reach the large, residual market share.

Quantitative data includes the market shares of the two products and the fixed parameters in the model (5)–(6).

4.2. Control of Marketing Activities. In order to obtain results that can be compared, all parameters of the system of differential equations (5)–(6) are kept constant, except the parameter α. This means that the coefficients of external influence a and p, coefficients of internal influence b and q, and nonuniform influence parameter β are fixed for the simulation, while the nonuniform influence parameter α is controlled by our fuzzy controller. The parameters a, b, p, q, β were chosen from the range of values which were found in previous empirical studies for different products (Refs. 1, 5, 11). In describing the simulation, the market share of the company product is denoted by P_1 and that of the competitor product by P_2.

The simulation is terminated if one market share is equal to zero or if the sum of both market shares is equal to one. If the first occurs, the model changes, because there is no influence from one product to the other anymore, and the simulation stops. If the second case occurs, the model cannot describe the diffusion process any longer, since

$$dP_1(t)/dt = 0 \quad \text{and} \quad dP_2(t)/dt = 0$$

in our proposed diffusion model (5)–(6); i.e., the market shares cannot change after that point.

The fuzzy logic controller can successfully control the diffusion process in many situations, but its success depends on the parameters that describe the model, and above all on the linguistic rule base which is related to the prevalent managerial thinking and beliefs.

When the fuzzy controller is successful, we mean that the market goal is achieved. This is done by changing α (marketing activities) with time within the constrains of the managerial belief system used for the simulations. In that case, the diffusion process converges to a stationary solution and the nonuniform influence parameter α remains approximately constant after a certain time; i.e., the two products split the market, and there are only marginal changes of less than 1% in the market shares for both products. Failure of the fuzzy controller indicates that one could not adjust α within the constrains of the managerial belief system to reach the market goal.

4.3. Efficiency Index. The fuzzy control which includes managerial thinking is exercised by varying the internal coefficient of influence α to control product penetration. In order to assess the results of the simulation, one needs to correlate the amount of resources committed to promoting a product (marketing activities) with α. In fact, we can introduce an efficiency index, I_e, defined as

$$I_e = \int_0^T f(\alpha)\, d\alpha, \tag{9}$$

over a certain time interval $[0, T]$. The functional dependence $f(\alpha)$, though extant, needs further exploration through empirical data. We therefore make a crude simplifying assumption that resources spent are inversely proportional to α. This makes

$$I_e \approx \int_0^T (1/\alpha)\, d\alpha. \tag{10}$$

Accordingly, we measure I_e in all the examples provided in this paper; since we use a piecewise constant α, the integral is reduced to a simple summation; thus here,

$$I_e = \sum_i 1/\alpha_i, \tag{11}$$

where α_i is the value in the ith time period. A high value of the efficiency index I_e eventually means more resources need to be spent to influence the product market penetration vis-a-vis its competitor.

Such a measure is admittedly rather crude. Obviously, it needs to be used along with other criteria, such as availability of initial capital outlays, time criticality of introduction of a product, etc. Several of these may be dependent on the specific industry.

4.4. Simulation Results. To illustrate the usefulness of our approach, we present several simulations in some detail. First, five different situations where the underlying dynamics is governed by the same set of equations were considered. Values for the parameters in all five situations were chosen as

$$a=0.0197, \quad b=0.4004, \quad p=0.0122, \quad q=0.4772, \quad \beta=1.13.$$

The nonuniform influence parameter α was allowed to vary between 0.1 and 12; see Fig. 7.

The NUI model (5)-(6) is highly nonlinear, and the efficacy of any strategy would depend on the parameters chosen to describe the model dynamics. Therefore, we also considered two additional examples where only one of the parameters (q) was changed from the previous situations. Nevertheless, we observed markedly different outcomes in market penetration generated by controlling α.

(i) Examples 4.1 and 4.2 deal with successfully controlled processes in the competition scenario.
(ii) Examples 4.3 and 4.4 deal with successfully controlled processes in the antimonopolistic scenario. Stabilization of the results takes longer compared to Examples 4.1 and 4.2.
(iii) Example 4.5 deals with the introduction of a new product in the competition scenario, successful for the aggressive strategy, but not successful for the conservative strategy.
(iv) Example 4.6 is the same as Example 4.2, but with $q=0.6804$. The introduced product is driven out of market while using the conservative strategy.
(v) Example 4.7 is the same as Example 4.4, but with $q=0.6272$. The diffusion process using the conservative strategy exhibits a very smooth behavior, while the relative efficiency of both strategies changes significantly.

In all the examples, the solid line in the first plot represents the course of the market share P_1 for the company product, while the dashed line describes the course of the market share P_2 of the competing product. The second plot exhibits the computed values for the inverse of the nonuniform influence parameter α used to compute the efficiency factor I_e, the estimate of resources committed.

We assume in all examples that the marketing activities are revised every three months, and that they cannot be changed in between.

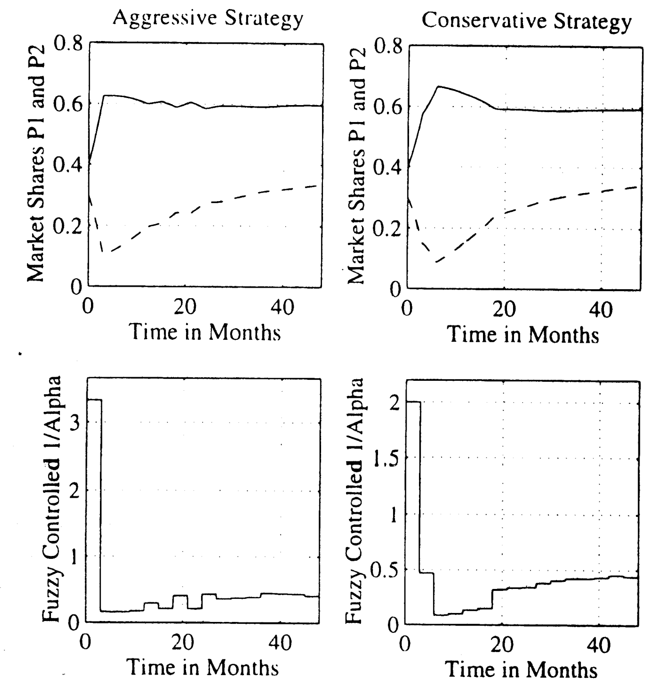

Fig. 8. Example 4.1 in competition scenario. Goal for Product 1: 0.6; initial market share for Product 1: 0.4; initial market share for Product 2: 0.3.

Examples 4.1 and 4.2. Both examples show a diffusion process in the competition scenario.

In Example 4.1, the goal of the company is to reach a market share of 60%, starting from a market share of 40%. The competing product has an initial market share of 30%. The results of the simulation are shown in Fig. 8.

Employing the aggressive strategy, the market share increases to approximately 60% in three months, i.e., after one decision period. Afterward, the company goal is to keep its market share at this level. The controller succeeds in reaching this goal, and Fig. 8 shows that there are only small oscillations around the desired market share $P_1=0.6$. Meanwhile, the market share P_2 of the competing product increases slowly toward the residual market share of 40%.

Using the same initial conditions and the conservative policy, it takes one more decision period (i.e., three more months) to reach the goal. Efficiency of both strategies, as measured by the index I_e, differs by less than 20% (see Table 1); thus, from this simulation, it is difficult to conclude which

Table 1. Efficiency indices for Examples 4.1 to 4.7.

Efficiency index	Aggressive strategy	Conservative strategy	$\frac{I_e(\text{aggr}) - I_e(\text{cons})}{I_e(\text{cons})}$
Example 4.1	8.20	6.87	19.4%
Example 4.2	14.12	9.73	45.1%
Example 4.3	7.32	6.73	8.8%
Example 4.4	14.71	8.68	69.5%
Example 4.5	43.69	*	—
Example 4.6	20.81	*	—
Example 4.7	17.74	29.89	—

strategy is more efficient, the aggressive policy with higher initial influence (lower α value) and lower influence afterward (higher α value), or the conservative strategy with lower initial influence, but higher influence in the later stages of the process.

In Example 4.2, we changed the initial conditions to the company starting market share of 30%, while the competing product has an initial market share of 40%. The results are similar to those observed in Example 4.1. Here, the diffusion process, controlled by the fuzzy logic controller using the aggressive strategy, reaches earlier the desired market share $P_1 = 0.6$, but the controller overshoots to about 70% of the market share for quite a long time. This is then reflected in a higher cost estimate; see Table 1.

Examples 4.3 and 4.4. In the antimonopolistic scenario, the goal of the company is to allow the competing product only a certain market share, while the company own product should reach a larger residual market share.

In Example 4.3, the company wants to get the competing product to a market share of $P_2 = 0.2$ and to retain it there. Figure 9 exhibits the results of the simulation. The fuzzy logic system was able to control the diffusion process using both the aggressive strategy and the conservative strategy. The observations made in Examples 4.1 and 4.2 in the competition scenario also hold for the antimonopolistic scenario.

However, we observe one major difference between the results in both scenarios. Compared to the competition scenario, it takes longer to get sufficiently close to the company goal and then to retain it. The oscillations in the market share P_2 last longer and have a higher amplitude. Similarly, more apparent are the higher oscillations in the values of the nonuniform influence parameter α. The efficiency index I_e is about the same for both strategies; see Table 1.

In Example 4.4, the company competes with a product which has an initially higher market share than the company's product. The goal is to get

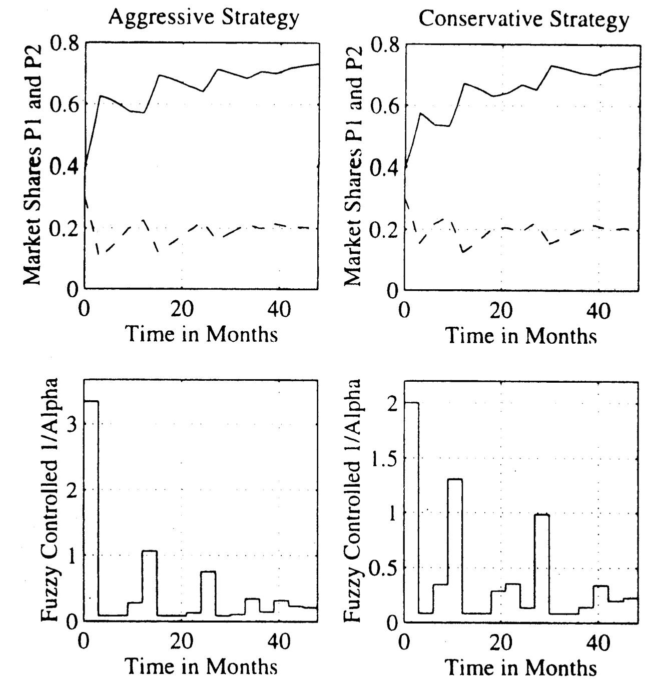

Fig. 9. Example 4.3 in antimonopolistic scenario. Goal for Product 2: 0.2; initial market shares for Product 1: 0.4; initial market share for Product 2: 0.3.

the competing product from a market share of 40% to a market share of 20%. The company own product has an initial market share of 30%.

Example 4.5. Here, we study again a diffusion process in the competition scenario. The difference with the earlier examples is that the company tries to introduce its product to the market, while the competing product is already established. Employing the aggressive strategy, Fig. 10 shows that the company succeeds in introducing its product. The simulation also demonstrates that the company is not successful in introducing its product into the market if it uses the conservative strategy. Already after three months (i.e., after one decision), the product is driven out of market (not shown in Fig. 10). This shows that, with the prevalent conservative strategy, the product cannot be successfully launched. The cost of introducing a new product is, as expected, quite high; the efficiency index I_e is about three times higher than in all other cases; see Table 1.

The behavior of the diffusion process employing the aggressive strategy points out some interesting aspects. After the first decision period, the market

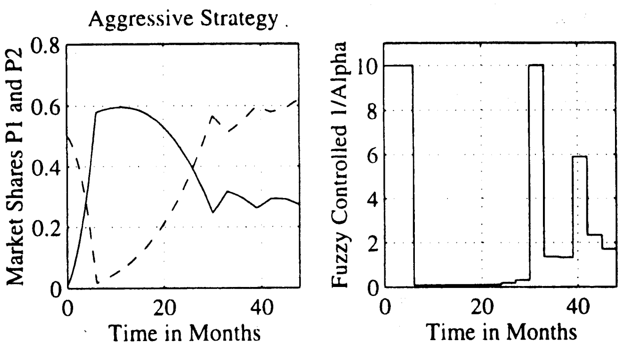

Fig. 10. Example 4.5 in competition scenario. Goal for Product 1: 0.3; initial market share for Product 1: 0.01; initial market share for Product 2: 0.5.

share P_1 increased from 1% to 20%, because the company used the maximum possible amount of resources (i.e., the value of influence $\alpha = 0.1$). In planning the influence α for the second step, already 2/3 of the desired market share of 0.3 has been achieved. In spite of this, the controller (as implemented by now) decides to use $\alpha = 0.1$ again, which leads to an overkill. As a result, P_1 reaches the market share of 0.58, well over the goal. After this step, the system changes the influence coefficient from one extremum α value of 0.1 to the other extremum value $\alpha = 12$, and the market share slowly oscillates to the desired value. This implies that the controller should slow down after the first step. This can be achieved by implementing an adaptive fuzzy rule base and hence an adaptive controller.

Example 4.6. In this example, the scenario and the initial conditions are the same as in Example 4.2, but with the parameter q changed to $q = 0.6804$. The aggressive strategy produces basically the same results as before. Using the conservative strategy, the market share takes a sharp dive down, and within a year is driven out of the market. This is the consequence of committing insufficient resources. Predictions such as these, which incorporate linguistic thinking, may be valuable tools to guide managerial policy.

Example 4.7. In this example, the scenario and the initial conditions are the same as in Example 4.4, but with the parameter q changed to $q = 0.6272$. In Fig. 11, one can observe a very smooth behavior of the diffusion process using the conservative strategy. The reason for the smoothness of the graphs is that the values for α remain between 0.4 and 0.5 for 30 months; i.e., the company needs to exercise high or very high influence steadily for 10 decision periods. As soon as the competing product market share reaches

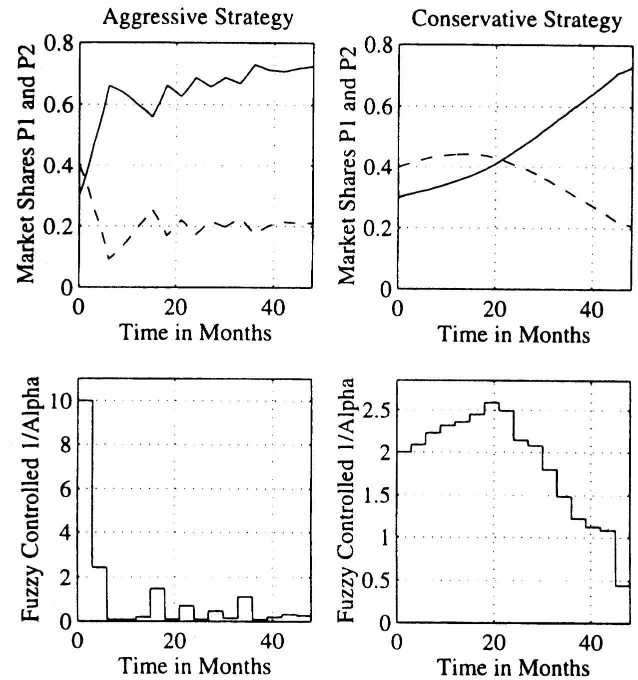

Fig. 11. Example 4.7 in antimonopolistic scenario. Goal for Product 2: 0.2; initial market shares for Product 1: 0.3 and for Product 2: 0.4. Membership functions for the market share.

the desired goal of $P_2 = 0.2$, the value of α increases; i.e., the company reduces the influence exercised on the market.

This example also demonstrates that we can avoid oscillations if the controller chooses values of α which do not change drastically. In real life, a marketing activity does not lose its effect as soon as the company decides to discontinue this specific marketing activity; rather, it shows a constantly declining effect. This observation leads to another possible refinement of the fuzzy logic controller by implementing a defuzzifier that should be able to include the decreasing effect of marketing activities over time.

4.5. Estimate of Resources. Table 1 reports the efficiency indices for Examples 4.1 to 4.7; the asterisk, denotes that introduction of the product was unsuccessful; see also Figs. 8–11. Values for the parameters in Examples 4.1 to 4.5 were

$$a = 0.0197, \quad b = 0.4004, \quad p = 0.0122, \quad q = 0.4772, \quad \beta = 1.13.$$

The results of Examples 4.1 to 4.5 in Table 1 may suggest that the conservative strategy is perhaps more cost effective (I_e is larger for the aggressive strategy by 10% to 70%), in all cases when it works. Yet, dealing with the nonlinear system (5)-(6), generalizations are difficult to make as illustrated by Examples 4.6 and 4.7. Here, only one parameter (q) was changed from Examples 4.2 and 4.4. Nevertheless, the results are significantly different; in Example 4.6, the introduction of the product using the conservative strategy was unsuccessful; in Example 4.7, this strategy becomes less efficient than the aggressive strategy.

In Example 4.5, the introduction of a new product requires significantly more marketing activities than that required to increase the market share of an established product, as in Examples 4.1 to 4.4.

5. Conclusions

In this paper, we have provided an approach to use qualitative data (managerial beliefs and strategies) and quantitative data to predict whether or not certain corporate goals can be achieved. The approach enables us to compare the efficacy of different strategies. While this general approach can be used for different kinds of ill-structured problems, we have confined our attention in this work to an application of market penetration in a two-product world. While studies such as these are crude approximations of real-life situations, they may still be well worth undertaking. They may shed light on whether certain corporate goals are achievable within the confines of given types of managerial thinking.

The diffusion model which we have used to illustrate our approach may be considered as a tool for forecasting market penetration. A company can influence the diffusion process via its decisions about marketing activities. We have shown that, by incorporating appropriate qualitative and quantitative data, strategies, and scenarios, one can predict and perhaps control the market penetration under given market conditions. Simulations show that one can use less resources for certain market conditions to reach a desired goal, and one can estimate the resources needed. Our approach can thus serve as a decision support tool for management wanting to implement a particular strategy. Simulations have also shown that under certain strategies and conditions the desired market penetration goal may not be achievable.

Though the prediction and control of market penetration is an important issue in itself, the general methodology presented herein can be used to forecast and control the outcomes of models, while including linguistically stated data and strategies, in other areas of decision support as well.

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