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GLOBAL PATTERNS FROM LOCAL INTERACTIONS: A DYNAMICAL SYSTEMS APPROACH

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In this paper the global patterns that result from local interactions between players on a two-dimensional lattice are studied. The assumptions on interaction between players are based on the Prisoner's Dilemma game that has been used extensively in game theory and in the study of biological systems. Each player is located on a square lattice, and is assumed to cooperate or defect, based on mimicking the neighbor with the highest cumulative score from the preceding round of play. The edges of the lattice are glued to form a torus. Computer simulations are conducted for different sized lattices, different payoff values, and different initial conditions. Though the paper is primarily concerned with player behavior without self-interaction, some results with self-interaction are also included. The influence of "ideal" cooperators on the evolution of the system dynamics is also studied. Three generic regimes of behavior are identified. Complex global patterns with complicated dynamics and sometimes unpredictable results occur. Steady-state solutions, simple and complex periodic solutions, and traveling waves are observed depending on the initial conditions and the payoff values.

Keywords: Prisoner's Dilemma; game theory; bifurcation; cooperator; defector.

1. Introduction

The notion of the importance of patterns is perhaps as old as civilization itself. Every art is founded on the study of pattern. The cohesion of social systems depends on the maintenance of patterns of behavior, and advances in civilization often depend on the modifications of such behavioral patterns. And yet these global patterns of behavior in a system emerge from the myriads of local interactions that occur among its participants. Most of these local interactions are nonlinear, and such analyses have, for the most part, been beyond the scope of the available analytical tools of mathematics. It is only with the advent of the computer that we have begun to investigate the emergence of these global patterns and their dependence on local interactions. This paper is a contribution towards this investigation.

Most modern societies are governed by mores, codes of conduct and laws. The peaceful coexistence of individuals usually requires that they be considerate of each other and desist from taking undue advantage of each other, through, say, participation in criminal, illegal or unacceptable behavior. Consider the example of a single criminal in a community. Suppose b is a dimensionless parameter that represents the perceived gain (profit) derived from engaging in criminal activity, taking into consideration factors like, the probability of being detected, possible magnitude of punishment, etc. If the criminal exploits his neighbors and makes a large profit, will he spark a crime wave with other people following in his footsteps? Does the percentage of people who follow in his footsteps over the long haul necessarily increase as the gains, b, from such criminal activity keep increasing? If a crime wave is initiated, how will it travel through the community? Will a steady-state, periodic, or chaotic crime-dynamic develop? Would the presence of a few "upright" individuals who cannot be swayed by profit cause criminal behavior to be curbed in a community?

Questions similar to these can also be raised in an ecological context about the competitive behavior of two species of animals that cohabit a given tract of land; or, about the proliferation of terrorist cells in a community where the "gain" from such malevolent behavior is measured by the disruption/fear that can be caused in a politically polarized environment. All one needs do is replace the word "criminal" with "terrorist" in the afore-mentioned set of questions in the previous paragraph.

Can the dynamics of such a complicated system be mathematically modeled and predicted? Is it possible to stimulate (control) such a dynamical system so that it achieves a stable, desired state? These are some of the questions that have motivated this study. To begin developing insight into this type of problem, we choose to explore the nonlinear dynamical response of a two-dimensional Prisoner's Dilemma game, similar to the one proposed by Nowak *et al.* [Nowak & May, 1992; Nowak *et al.*, 1993, 1994a, 1994b, 1995a, 1995b]. Some of the essential aspects of such a nonlinear dynamic system are determination of the absorbing sets, the transient behavior towards the absorbing sets, and their dependence on the number of players in the lattice.

While the majority of the more recent studies of the iterated Prisoner's Dilemma appear focused on biological systems, our interest is on the application of the iterated Prisoner's Dilemma to the social, political and economic structures of human civilization. The success of human society depends on a high level of voluntary, and sometimes forced, cooperation. Crime rates rise and fall; the economy has good times and bad times; political agendas shift; businesses merge, and then split. One possible way of modeling these expanding and contracting patterns of "cooperation" and "defection" may be the iterated Prisoner's Dilemma type of game.

The paradox of the Prisoner's Dilemma embodies the struggle between cooperation and exploitation. Though known to the ancient Greeks, the Prisoner's Dilemma was precisely formulated in the 1950s, and in its classical form refers to two prisoners involved in a crime, and the decision each must make to either cooperate with the other or to defect by cooperating with the authorities [Nowak *et al.*, 1995a, 1995b]. Neither player knows in advance what the other will do, and the severity of the punishment depends on their decision.

Traditionally, the Prisoner's Dilemma has been viewed as an iterated game between *two* players. If both players cooperate then each gets a reward, R, for mutual cooperation. If both players defect then each gets a punishment, P, for mutual defection. If one player defects and the other cooperates, the gain for defection, T, is awarded to the defector, and the payoff, S, is awarded to the cooperator [Axelrod, 1984]. The total score for each player is the cumulative score from each round. With $T > R > P > S$, the best strategy for each player in any given round is to defect. This leads to the gain T if playing against someone who cooperates, and P if playing against someone who defects. Alternatively, the strategy of cooperating will at best lead to award R if playing someone who also cooperates, but will result in the payoff S if playing someone who defects. See Fig. 1. The dilemma exists because the strategy of defecting is unbeatable relative to your opponent's score, but when both players think this way and therefore defect, then each receives less than they would have, had each player cooperated.

The Prisoner's Dilemma game has been studied extensively in the context of cooperation theory. Computer tournaments have been conducted to determine the best strategy for success in the iterated Prisoner's Dilemma. One of the simplest and most successful strategies was Tit For Tat (TFT), whereby the player starts out cooperating, and subsequently does what the opponent did on the previous move [Axelrod, 1984]. Because the strategy depends only on the opponents move in the previous

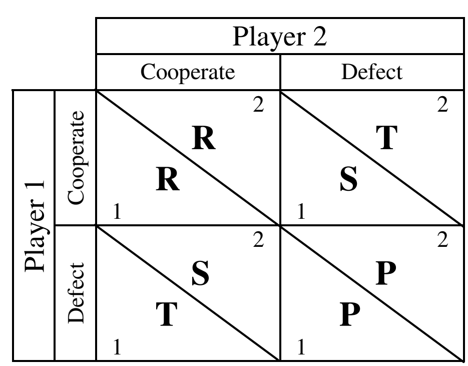

Fig. 1. Payoff matrix for the Prisoner's Dilemma game.

round, this is a "memory-1 strategy." Variations of TFT, such as Generous Tit For Tat (GTFT) and "Pavlov" have also been shown to be very successful. GTFT players generally copy their opponent's last move, but occasionally cooperate after their opponent defects. "Pavlov", also known as "win-stay, lose-shift," is a memory-2 strategy that depends on both players previous move [Milinski & Wedekind, 1998].

More complicated strategies, and variations to the iterated Prisoner's Dilemma, have been proposed in an effort to more accurately model different systems. These strategies have included extended memory of previous encounters, and complex algorithms to try and anticipate the opponent's next move. A memory-4 strategy with random mutations has been proposed as a model of primate behavior [Key & Aiello, 2000]. Spatial mobility of various types of players has been included in some models [Ferriere & Michod, 1995; Hutson & Vickers, 1995]. If the payoff for each encounter is allowed to be variable, then a strategy of "raise-the-stakes" offers insight into the development of cooperation [Roberts & Sheratt, 1998].

Rather than the traditional game between two players, our model consists of a two-dimensional square lattice of stationary players with dimensions $n \times n$. The edges of the lattice wrap around in the shape of a torus, forming periodic boundary conditions so that the players form a "closed community." Each player competes against each of his eight immediate neighbors during each round of the game. At the end of each round of play, each player sums up his gains from having played against his eight immediate neighbors on the lattice. Gains are defined as $T = b$, $R = 1$, and $S = P = 0$, thus preserving the essential paradox in the Prisoner's Dilemma, while simplifying the computations and the understanding. Further, we assume a "follow the leader" type strategy wherein each player chooses to either cooperate or defect by following the strategy of the neighbor with the maximum gain from the previous round. While this assumption may appear an oversimplification, some experimental data has shown that human beings tend to choose such strategies while playing the Prisoner's Dilemma game [Milinski & Wedekind, 1998; Wedekind & Milinski, 1996], thus this simplification may perhaps not be too unrealistic.

Nowak *et al.* [Nowak & May, 1992; Nowak *et al.*, 1993, 1994a, 1994b, 1995a, 1995b] have reported results from a similarly formulated iterated Prisoner's Dilemma game. Their work considered both fixed and periodic boundary conditions, but focused primarily on situations in which each player plays with his immediate neighbors *and* with himself. Thus their results mainly concern games *with* self-interaction.

2. Global Behavior from Local Interactions

Since one of our motivations is the need to understand the system dynamics engendered by criminal/social behavior, in this paper we concern ourselves primarily with games without self-interaction. We also provide in-depth results on the detailed transient dynamics, and give a useful categorization of the global dynamics into three regimes of behavior. There are several different specific aspects of this problem that we report on:

(a) analytical determination of the possible bifurcation points of the dynamical response;
(b) study of the dynamic response versus the parameter b for the simple "initial condition" of a single defector in the center of a square lattice;
(c) effects on the dynamical response of increasing the lattice size;
(d) sensitivity of the dynamics to small variations in the initial conditions of symmetrically placed initial defectors;
(e) characterization of the system dynamics when the initial condition is a random distribution of initial defectors;
(f) consideration of self-interaction, solely for comparison purposes, when starting from random initial distributions of defectors;
(g) influence of including ideal cooperators — individuals who will not defect no matter what their gains — on the global patterns; and,
(h) discussion on the use of periodicities and percentages of defectors as metrics for understanding the global dynamics.

The type of model studied in this paper is also relevant to ecological dynamics [Dieckmann *et al.*, 2000]. It can be viewed as an *extension* of the so-called "lattice gas models" in physics and engineering [Doolen, 1991] in which particle interactions are modeled to occur on a lattice or regular grid, and the laws of interaction now go beyond the usual physical laws of mass, momentum and energy balance. For example, a simplified game where the players are updated in random sequence and have a chance to adopt the neighboring strategies with a probability depending on the payoff has been investigated [Szabo & Toke, 1998]. A tit-for-tat strategy [Szabo *et al.*, 2000] can be included in addition to cooperation and defection. While most studies of this kind simply report computational results, the in-depth analysis performed here allows us to categorize and understand the qualitative behavior behind the generation of the multitude of dynamical behaviors exhibited.

In what follows, for convenience of representation, players are plotted in different colors to indicate their previous and next decisions, either to cooperate or to defect. A blue player cooperated on the previous game, and cooperates again in the next. A red player defected on the previous game, and defects again in the next game. A green player defected on the previous game, but cooperates in the next game. A yellow player cooperated on the previous game, but defects in the next game. Ideal cooperators are shown in magenta colored asterisks.

(a) Analytical determination of the possible bifurcation points of the dynamical response.

The qualitative nature of the system dynamics depends on the value of the parameter b — the gains of defection. The value of b where the qualitative dynamics changes is defined as a bifurcation point. Because of the deterministic nature of this iterated Prisoner's Dilemma game, a finite number of discrete bifurcation points exist. The possible bifurcation points may be calculated by considering the total possible payoffs to a cooperator, and to a defector, as indicated below in Table 1. A cooperator will score 1 point from each neighboring cooperator and 0 points from each neighboring defector, thus his total score from any given round may range from 0 points to 8 points (excluding self-interaction). A defector will score b points from each neighboring cooperator and 0 points from each neighboring defector, thus his total score from any given round will be a multiple of b, ranging from 0 to $8b$. Thus, bifurcation points *may* occur when the score of a cooperator exactly matches that of a defector. In the course of our numerical simulations, we found that bifurcation points often coincide with *asymmetric* expansion or contraction of clusters of defectors and cooperators, when starting with a single initial defector. The bifurcation points given in Table 1 were validated by methodically varying the value of the payoff parameter, b, during the numerical simulations we conducted. Bifurcation values of b less than 1.0 are of little importance for our purposes, as this implies that the payoff (gain) for defecting is less than the payoff for cooperating. With no incentive to defect, players will generally cooperate.

Table 1. Possible discrete values of bifurcation points for games played without self-interaction. Values less than 1.0 are of little importance for our study because with no incentive to defect, players will generally cooperate.

Bifurcation Points, *b*		Defector Payoff							
		8*b*	**7*b***	**6*b***	**5*b***	**4*b***	**3*b***	**2*b***	**1*b***
Cooperator Payoff	**8**	8/8	8/7	8/6	8/5	8/4	8/3	8/2	8/1
	7	7/8	7/7	7/6	7/5	7/4	7/3	7/2	7/1
	6	6/8	6/7	6/6	6/5	6/4	6/3	6/2	6/1
	5	5/8	5/7	5/6	5/5	5/4	5/3	5/2	5/1
	4	4/8	4/7	4/6	4/5	4/4	4/3	4/2	4/1
	3	3/8	3/7	3/6	3/5	3/4	3/3	3/2	3/1
	2	2/8	2/7	2/6	2/5	2/4	2/3	2/2	2/1
	1	1/8	1/7	1/6	1/5	1/4	1/3	1/2	1/1

(b) Study of the dynamic response versus the parameter b for the simple "initial condition" of a single defector in the center of a square lattice.

The dynamic response is determined as a function of the value of the payoff parameter, b, for the case of a single initial defector in the center of a 29×29 square lattice without self-interaction, and with periodic boundary conditions. Through methodical simulations, it is determined that bifurcation of the dynamic patterns occur at b values of 7/8, 1, 6/5, 7/5, 8/5, 5/3, 7/4, 2 and 8/3. The different dynamical regions consist of 1-period, 2-period and 3-period solutions. Specific results for each dynamical region are detailed in Table 2 and Figs. 2 and 3. The regions are lettered A through J for convenience.

A critical bifurcation point exists at $b = 8/5$. Below $b = 8/5$, only the immediate neighborhood of the initial defector is influenced and changes states. Above $b = 8/5$, the region of defectors expands beyond the immediate neighborhood of the initial defector. The dynamics for $8/5 < b < 5/3$ (region F) are particularly interesting because clusters of defectors and clusters of cooperators dynamically expand, collide and collapse. Numerous games are usually required to reach the attracting state within this region.

(c) The effects on the dynamical response of increasing the lattice size.

The effects on the bifurcation points, spatial pattern, and transient dynamics, of increasing the

Table 2. Dynamical regions for a square lattices with dimensions n = 19, 20, 29 and 59, where the initial condition is a single defector in the center, there are periodic boundary conditions, and there is no self-interaction. Three distinct regions occur. For $b < 8/5$ (regions A through E), the dynamic response is localized about the initial defector and may be periodic or steady-state. The percentage of defectors in the absorbing state remains less than 2.5%. Region F ($8/5 < b < 5/3$) is a much more dynamically active region with expanding and contracting areas of cooperators and defectors, that ultimately results in a steady-state solution. For $b > 5/3$ (regions G through J), the region of defectors expands to wrap around the torus, and then quickly converges to a steady-state solution. Values denoted with an asterisks ($*$) indicate the number of games completed when the first pattern in a periodic solution is formed.

Region	Dynamic Range	Lattice Size	Games to Attracting State	% Defectors in Attracting State(s)	Description of System Dynamics
A	$b < 7/8$	19	1	0.00	The initial defector turns cooperator after the first game. The final state is all cooperators.
		20	1	0.00	
		29	1	0.00	
		59	1	0.00	
B	$7/8 < b < 1$	19	0	0.28	The initial defector remains as the steady-state 1-period solution. None of the players change states.
		20	0	0.25	
		29	0	0.12	
		59	0	0.03	
C	$1 < b < 6/5$	19	1^*	2.49, 0.28	The immediate neighbors of the initial defector form a 2-period solution. One resulting pattern of the 2-period solution is a 3×3 cluster of defectors, and the other is a single defector.
		20	1^*	2.25, 0.25	
		29	1^*	1.07, 0.12	
		59	1^*	0.26, 0.03	
D	$6/5 < b < 7/5$	19	1^*	2.49, 1.39, 0.28	The immediate neighbors of the initial defector form a 3-period solution. The spatial patterns are: a 3×3 cluster of defectors; a cross-shaped pattern of defectors; and a single defector.
		20	1^*	2.25, 1.25, 0.25	
		29	1^*	1.07, 0.59, 0.12	
		59	1^*	0.26, 0.14, 0.03	
E	$7/5 < b < 8/5$	19	1	2.49	The immediate neighbors of the initial defector form a 3×3 cluster of defectors, which is the 1-period steady state solution.
		20	1	2.25	
		29	1	1.07	
		59	1	0.26	

Table 2. (*Continued*)

Region	Dynamic Range	Lattice Size	Games to Attracting State	% Defectors in Attracting State(s)	Description of System Dynamics
F	$8/5 < b < 5/3$	19	24	100.00	Clusters of defectors and cooperators expand, collide, and contract. Eventually, the attracting state is 100% defectors.
		20	25	100.00	
		29	111	100.00	
		59	2437	100.00	
G	$5/3 < b < 7/4$	19	9	77.84	The majority of the players eventually defect, leaving only isolated steady-state clusters of cooperators. The percentage of defectors in the attracting state increases as the lattice size increases.
		20	9	85	
		29	14	87.16	
		59	32	89.09	
H	$7/4 < b < 2$	19	10	44.60	The region of defectors expands in an 'X' shaped pattern. The attracting state is an 'X' shaped steady-state solution. The percentage of defectors in the attracting state decreases as the lattice size decreases.
		20	10	41.50	
		29	14	30.08	
		59	29	14.74	
I	$2 < b < 8/3$	19	9	100.00	The majority of the players eventually defect, leaving only isolated steady-state clusters of cooperators. The percentage of defectors in the attracting state does not follow a consistent pattern as in other regions.
		20	9	83.50	
		29	16	94.29	
		59	33	91.38	
J	$8/3 < b$	19	9	100.00	The cluster of defectors grows as a square wave front, until all the players are defectors.
		20	9	100.00	
		29	14	100.00	
		59	29	100.00	

lattice size is studied for the case of a single defector in the center of a square lattice without self-interaction and with periodic boundary conditions. Square lattices with dimensions $n = 19$, 20, 29 and 59 are studied. With these conditions, the bifurcation points of the payoff parameter, b, are independent of lattice size. While specific spatial patterns generally vary with lattice size, the qualitative nature of the spatial patterns within a specific range of b values are similar regardless of the lattice size. One indicator of the specific spatial patterns is the percentage of defectors in the attracting state. The percentage of defectors in the attracting states generally varies only slightly as the size of the lattice changes. It is the transient dynamics of the spatial evolution that appears to be most affected by changes in lattice size. One indicator of this is the number of games required to reach an attracting state. See Table 2.

Three interesting dynamical regions emerge. For $b < 8/5$ (regions A, B, C, D and E), the local spatial patterns and the number of games required to reach an attracting state are independent of lattice size because only the immediate neighbors of the initial defector are influenced and change states. For $8/5 < b < 5/3$ (region F), the spatial pattern always contains expanding and contracting clusters of cooperators and defectors, but the specific spatial patterns that emerge are very different and depend on the size of the lattice.

The number of games required to reach the attracting state increases very rapidly with the size of the lattice. For the lattice sizes evaluated, the attracting state in region F is always 100% defectors when the initial condition is a single defector at the center. For $b > 5/3$ (regions G, H, I, J), the attracting state is a steady state solution, and the number of games required to reach the attracting

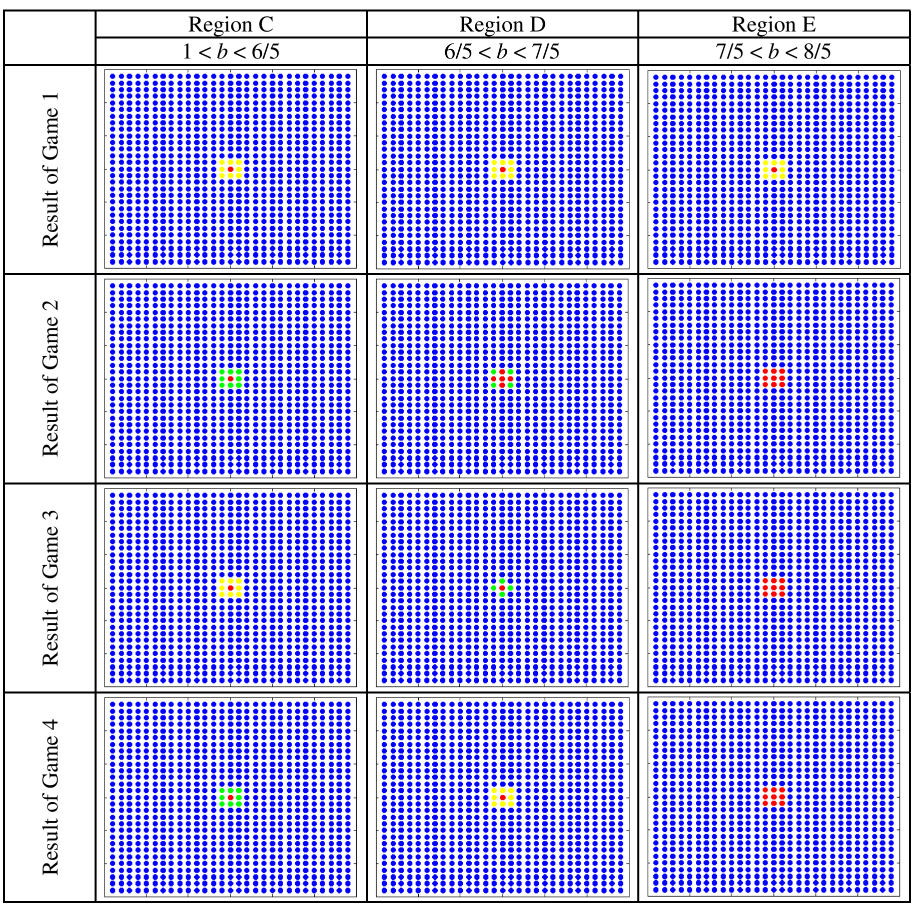

Fig. 2. Results of games 1–4 for dynamical regions between $b = 1$ and $8/5$ for an initial single defector at the center of a 29×29 lattice with no self-interaction and with periodic boundary conditions. A two-period solution exists in region C; a three-period solution in region D; and a one-period solution in region E. Regions A and B are not shown, as nothing particularly interesting occurs within these regions (see Table 2).

state increases linearly with the size of the lattice. See Table 2, and Figs. 4 and 5 for details.

(d) Sensitivity of the dynamics to small variations in the initial conditions of symmetrically placed initial defectors.

Sensitivity to initial conditions is studied by making small changes to simple, symmetrical initial conditions of a 29×29 lattice with periodic boundary conditions and without self-interaction. The following initial conditions are considered:

A. A single defector in the center (denoted by "D");
B. Two adjacent defectors near the center (denoted by "DD");
C. Two defectors with a single cooperator between them (denoted by "DCD");
D. Two defectors with two adjacent cooperators between them (denoted by "DCCD");
E. Two defectors separated by three cooperators (denoted by DCCCD);
F. Two defectors separated by four cooperators (denoted by DCCCCD);
G. Two defectors separated by five cooperators (denoted by DCCCCCD);
H. Three defectors in a triangle, separated vertically and horizontally by a single defector (denoted by D/C/DCD).

The spatial dynamics within each dynamic region, especially the periodicity of the attracting state, appears to be very sensitive to the initial conditions. 1-period, 2-period and 3-period solutions are common for $b < 8/5$. The region with $8/5 < b < 5/3$ also shows other higher period solutions,

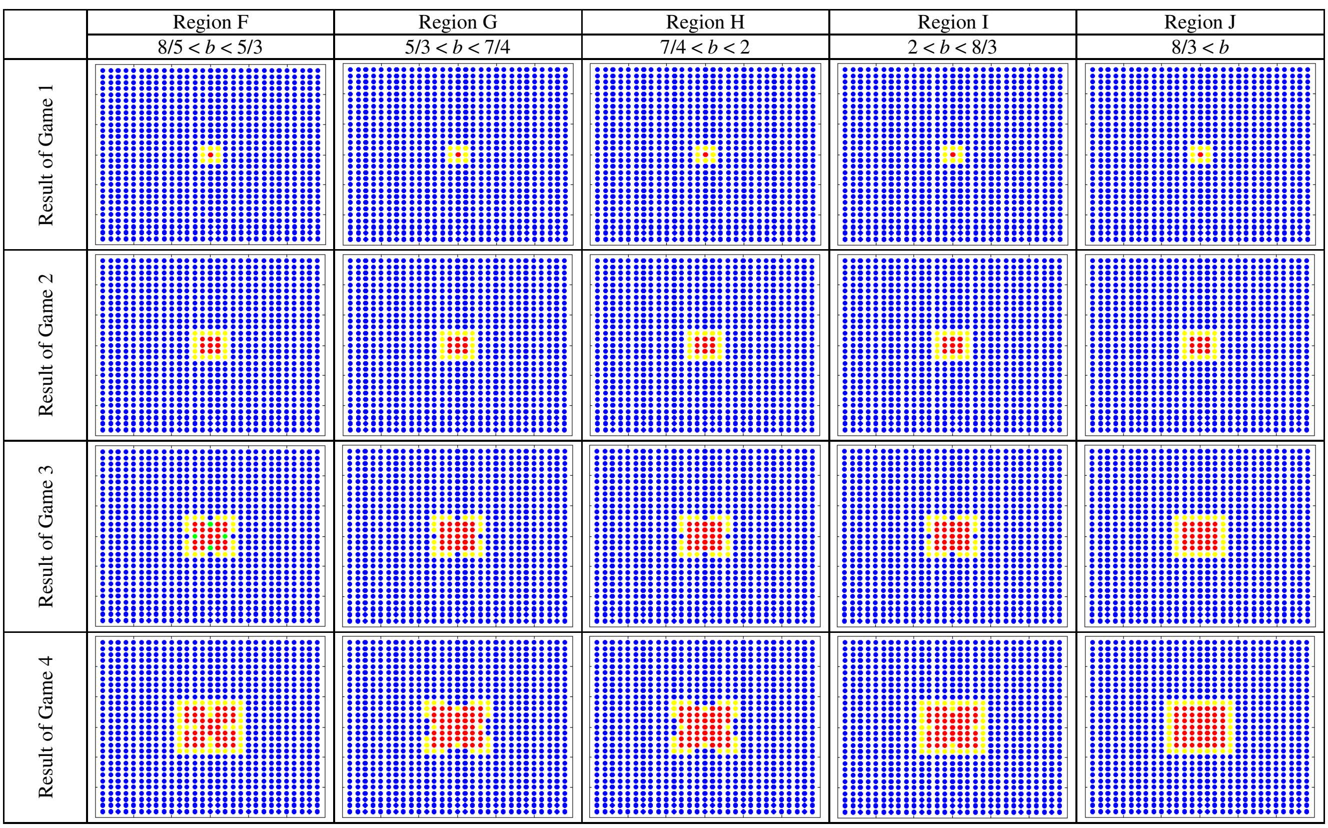

Fig. 3. Results of games 1–7 and 12–16 for the different dynamical regions with $b > 8/5$ for an initial single defector at the center of a 29×29 lattice with no self-interaction and with periodic boundary conditions. All five regions start similarly, but differences can start to be observed after the third game as bifurcations along the leading edge of each wave front start to occur. The most dynamic spatial patterns are generated in region F as clusters of cooperators and defectors begin to expand and contract. Regions G and I expand similarly, with the final attracting state dominated by defectors with only isolated clusters of cooperators. Region H expands as an "X" shaped wave front, and results in an "X" shaped one-period (steady-state) solution. Region J expands as a square wave front with no bifurcations along its leading edge, resulting in 100% defectors. Similar results occur for square lattices with dimensions $n = 19$, 20 and 59. See Table 2 for additional details on the spatial patterns.

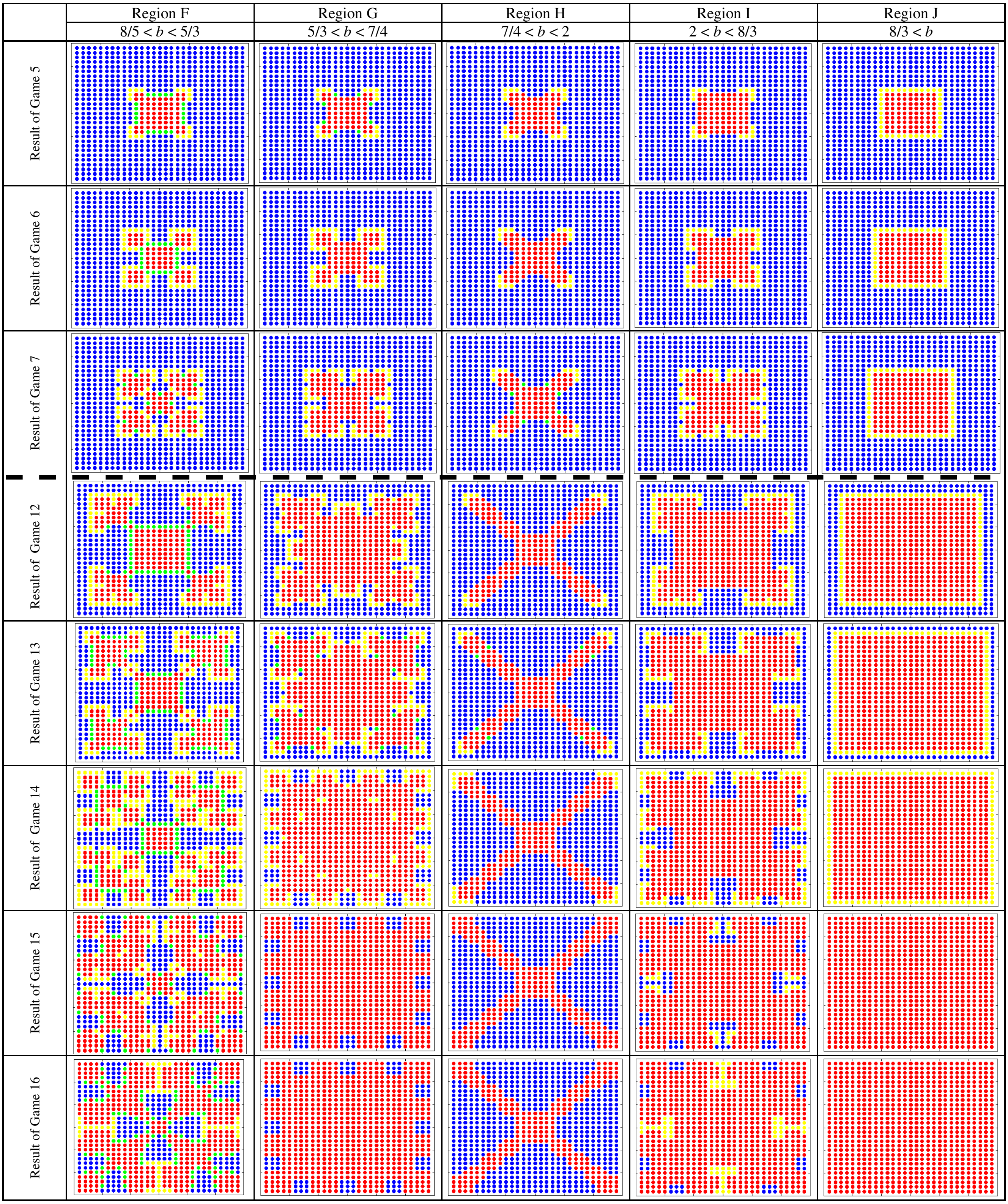

Fig. 3. (*Continued*)

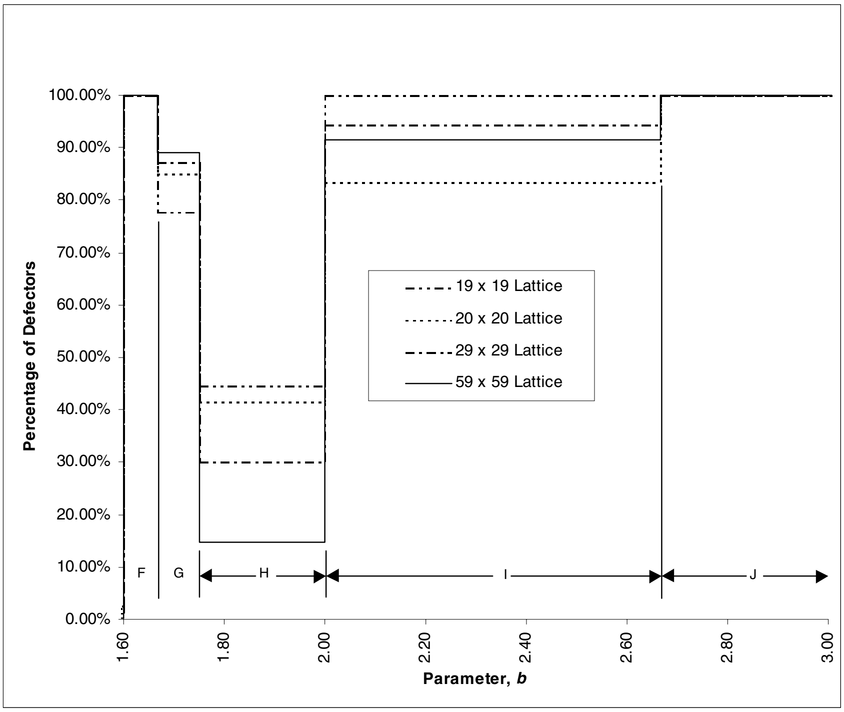

Fig. 4. The percentage of defectors in the attracting state is shown versus the parameter b, for lattices with dimensions $n = 19$, 20, 29 and 59, for the initial case of a single defector in the center, with periodic boundary conditions and without self-interaction. The percentage of defectors in the attracting state generally varies slightly with the size of the lattice. Note that the percentage of defectors in the attracting state seems to increase with lattice size in region G, and decrease with lattice size in region H.

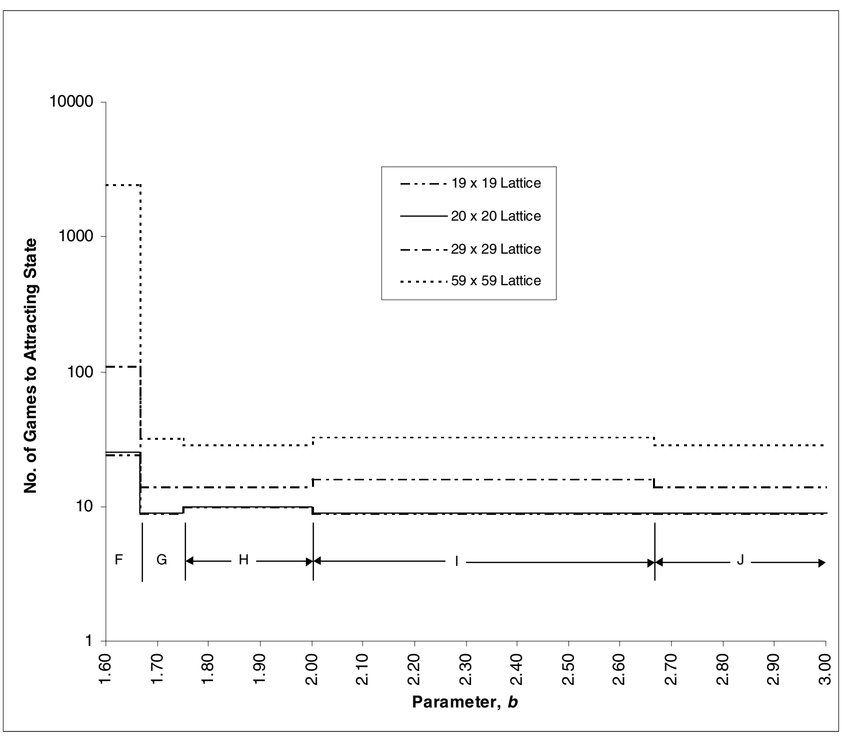

Fig. 5. The number of games required to reach the attracting state is shown versus the payoff parameter, b, for lattices with dimensions $n = 19$, 20, 29 and 59, for the initial case of a single defector in the center, with periodic boundary conditions and without self-interaction. The number of games required to reach the steady-state solution for $b > 8/5$ generally increases with the size of the lattice.

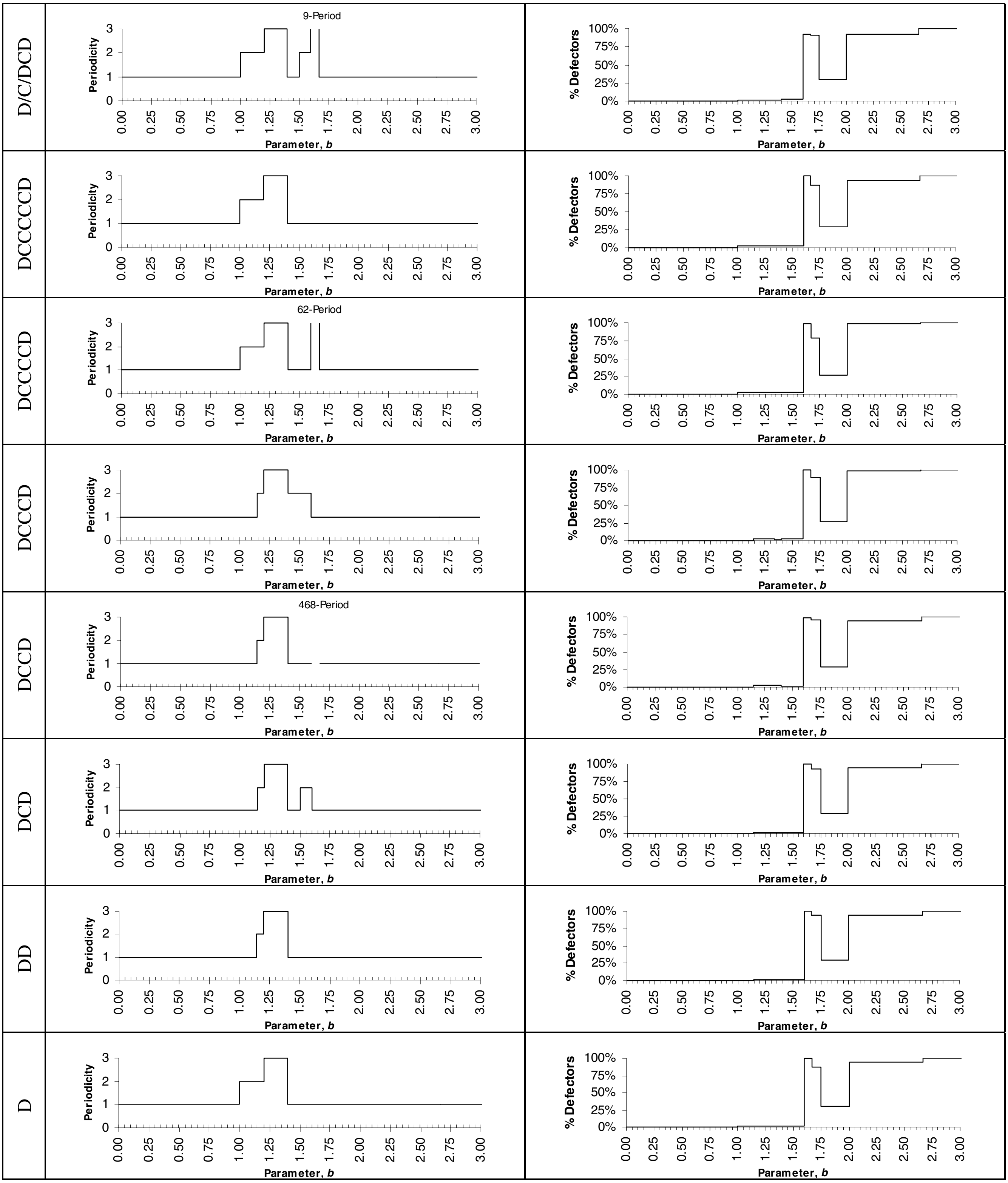

Fig. 6. Periodicities and percentage defectors in the attracting state resulting from different symmetric initial conditions for different values of b. Results are for a square 29×29 lattice with periodic boundary conditions, and without self-interaction. The periodicity is more sensitive to the particular initial conditions detailed herein than the percentage of defectors in the attracting state. One of the more complicated dynamic responses occurs for initial condition D/C/DCD; it consists of two distinct 2-period solutions, one 3-period solution and a 9-period solution, depending on the value of b. The periodicities of initial conditions D and DCCCCCD are identical. The five cooperators between defectors isolate the defectors sufficiently so that their influence remains localized and does not have a global effect. The percentage of defectors in the attracting state appears to be minimally affected by the change in initial conditions. (Note: For periodic attracting states, the maximum number of defectors in the attracting state is shown.)

Table 3. This table gives the periodicity ("Period") and maximum percentage of defectors in the attracting state ("% Def") for different values of gain, b, for small variations in symmetrical initial conditions. These simulations are conducted on a 29×29 lattice with periodic boundary conditions, and without self-interaction. Distinct regions of 1-period, 2-period and 3-period solutions were common. However, 9-period, 62-period and 468-period solutions also occurred. Not all bifurcation values in the payoff parameter, b, occur for each set of initial conditions. The bifurcation values that occur for each set of initial conditions are represented by the formatting of the cells in this table. Single cells indicate the regions over which the dynamics are identical.

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<tr><th rowspan="2"></th><th colspan="16">Initial Condition</th></tr>
<tr><th colspan="2">D</th><th colspan="2">DD</th><th colspan="2">DCD</th><th colspan="2">DCCD</th><th colspan="2">DCCCD</th><th colspan="2">DCCCCD</th><th colspan="2">DCCCCCD</th><th colspan="2">D/C/DCD</th></tr>
<tr><th>Parameter, b</th><th>Period</th><th>% Def</th><th>Period</th><th>% Def</th><th>Period</th><th>% Def</th><th>Period</th><th>% Def</th><th>Period</th><th>% Def</th><th>Period</th><th>% Def</th><th>Period</th><th>% Def</th><th>Period</th><th>% Def</th></tr>
<tr><td>$b < 7/8$</td><td>1</td><td>0.00</td><td rowspan="2">1</td><td rowspan="2">0.00</td><td>1</td><td>0.00</td><td>1 [2]</td><td>0.00</td><td>1</td><td>0.00</td><td>1</td><td>0.00</td><td>1</td><td>0.00</td><td>1</td><td>0.00</td></tr>
<tr><td>$7/8 < b < 1$</td><td>1</td><td>0.12</td><td>1</td><td>0.12</td><td>1 [2]</td><td>0.00</td><td>1</td><td>0.24</td><td>1</td><td>0.24</td><td>1</td><td>0.24</td><td>1</td><td>0.12</td></tr>
<tr><td>$1 < b < 8/7$</td><td rowspan="2">2</td><td rowspan="2">1.07</td><td>1</td><td>0.24</td><td>1</td><td>0.36</td><td>1</td><td>0.48</td><td>1</td><td>0.59</td><td rowspan="2">2</td><td rowspan="2">2.14</td><td rowspan="2">2</td><td rowspan="2">2.14</td><td rowspan="2">2</td><td rowspan="2">1.07</td></tr>
<tr><td>$8/7 < b < 6/5$</td><td>2</td><td>1.43</td><td>2</td><td>1.78</td><td>2</td><td>2.14</td><td>2</td><td>2.26</td></tr>
<tr><td>$6/5 < b < 4/3$</td><td rowspan="2">3</td><td rowspan="2">1.07</td><td rowspan="2">3</td><td rowspan="2">1.43</td><td rowspan="2">3</td><td rowspan="2">1.78</td><td rowspan="2">3</td><td rowspan="2">2.14</td><td>3</td><td>2.26</td><td rowspan="2">3</td><td rowspan="2">2.14</td><td rowspan="2">3</td><td rowspan="2">2.14</td><td rowspan="2">3</td><td rowspan="2">1.43</td></tr>
<tr><td>$4/3 < b < 7/5$</td><td>3</td><td>1.78</td></tr>
<tr><td>$7/5 < b < 3/2$</td><td rowspan="2">1</td><td rowspan="2">1.07</td><td rowspan="2">1</td><td rowspan="2">1.43</td><td>1</td><td>1.55</td><td rowspan="2">1</td><td rowspan="2">1.66</td><td rowspan="2">2</td><td rowspan="2">2.50</td><td rowspan="2">1</td><td rowspan="2">2.14</td><td rowspan="2">1</td><td rowspan="2">2.14</td><td>1</td><td>2.14</td></tr>
<tr><td>$3/2 < b < 8/5$</td><td>2</td><td>1.78</td><td>2</td><td>2.26</td></tr>
<tr><td>$8/5 < b < 5/3$</td><td>1</td><td>100.00</td><td>1</td><td>100.00</td><td>1</td><td>100.00</td><td>468</td><td>98.10</td><td>1</td><td>100.00</td><td>62</td><td>98.10</td><td>1</td><td>100.00</td><td>9</td><td>92.39</td></tr>
<tr><td>$5/3 < b < 7/4$</td><td>1</td><td>87.16</td><td>1</td><td>94.65</td><td>1</td><td>92.87</td><td>1</td><td>96.08</td><td>1</td><td>88.59</td><td>1</td><td>78.12</td><td>1</td><td>86.44</td><td>1</td><td>91.08</td></tr>
<tr><td>$7/4 < b < 2$</td><td>1</td><td>30.08</td><td>1</td><td>29.96</td><td>1</td><td>29.13</td><td>1</td><td>28.54</td><td>1</td><td>26.99</td><td>1</td><td>26.63</td><td>1</td><td>28.42</td><td>1</td><td>29.49</td></tr>
<tr><td>$2 < b < 7/3$</td><td rowspan="2">1</td><td rowspan="2">94.29</td><td>1 [1]</td><td>95.01</td><td rowspan="2">1</td><td rowspan="2">94.05</td><td>1 [1]</td><td>94.65</td><td rowspan="2">1</td><td rowspan="2">98.10</td><td rowspan="2">1</td><td rowspan="2">98.57</td><td rowspan="2">1</td><td rowspan="2">93.58</td><td rowspan="2">1</td><td rowspan="2">92.27</td></tr>
<tr><td>$7/3 < b < 8/3$</td><td>1 [1]</td><td>95.01</td><td>1 [1]</td><td>94.65</td></tr>
<tr><td>$8/3 < b$</td><td>1</td><td>100.00</td><td>1</td><td>100.00</td><td>1</td><td>100.00</td><td>1</td><td>100.00</td><td>1</td><td>100.00</td><td>1</td><td>100.00</td><td>1</td><td>100.00</td><td>1</td><td>100.00</td></tr>
</table>

Notes:

1. The attracting state for the regions and $7/3 < b < 8.3$ are identical. However, the attracting state is reached 1 game sooner for $2 < b < 7/3$.
2. The attracting state of all cooperators is reached after 1 game for $b < 7/8$, and after 3 games for $7/8 < b < 1$.

like 9-period, 62-period and 468-period solutions. Table 3 and Fig. 6 give our simulation results for periodicity as a function of the defector's gain, b.

The maximum percentage of defectors in the attracting state for each dynamic region, however, does not appear to be very sensitive to the symmetrical initial conditions we considered. The maximum percentage of defectors in the attracting states for the symmetrical initial conditions that are considered are given in Table 3, and graphically displayed in Fig. 6.

The response to different initial conditions is particularly interesting in the region $8/5 < b < 5/3$. Not only do attracting states with long periods exist within this region, but the transient dynamics and the number of games required to reach the attracting state are very sensitive to the initial conditions. We found that the number of games required to reach the attracting state generally increases with the complexity of the initial conditions. The number of games required to reach the attracting state, whether steady-state or periodic, varies from 73 to 4,690 for the initial conditions that we considered, as shown in Fig. 7.

(e) Characterization of the system dynamics when the initial condition is a random distribution of initial defectors.

As the previous analysis indicates, there appear to be three well-defined regions in the value of the parameter b when a simple, symmetrical initial condition is used. For $b < 8/5$, the affected spatial region is limited to the immediate neighborhood of the initial defector(s); attracting states are reached quickly and consist of steady-state or periodic solutions. For $8/5 < b < 5/3$, the attracting state generally takes many games to reach, and is either a steady-state solution (typically 100% defectors), or a periodic solution. For $b > 5/3$, the attracting state is a steady-state solution; no periodic solutions exist within this region.

These three regions are further explored in this section for the case of a random distribution of initial defectors in a square lattice without self-interaction and with periodic boundary conditions. Simulations are conducted with $b = 1.45$ or

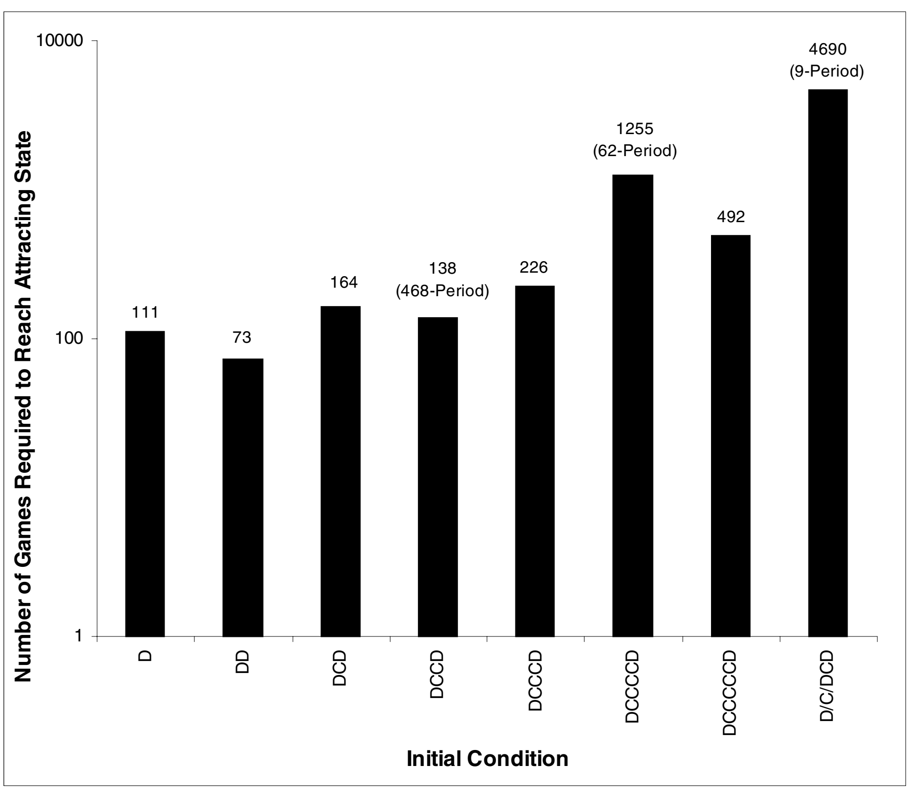


Fig. 7. Number of games required to reach the attracting state varies significantly for small variations in the initial conditions. Note that the vertical scale is logarithmic. These simulations were conducted for a 29×29 lattice with periodic boundary conditions and without self-interaction, with $8/5 < b < 5/3$.

Table 4. This table describes typical results achieved for simulations with gain values of $b = 1.55$, 1.63 and 1.70. The results are shown for a 20×20 lattice with periodic boundary conditions and without self-interaction. Similar results seem to occur regardless of lattice size. The percentage of defectors versus the number of games completed, and representative spatial patterns corresponding to these descriptions are shown in Figs. 8 and 9, respectively.

Initial % Defector	$b = 1.55$	Initial % Defector	$b = 1.63$	Initial % Defector	$b = 1.70$
1%	The attracting state is reached after 5 games and is a 2-period solution with localized clusters of defectors. Some clusters have players that alternate their states. In this case, the yellow players alternate yellow-green-yellow... (i.e. defector-cooperator-defector...)	1%	The percentage of defectors grows quickly in the first few games. The regions of cooperators and defectors each expand, contract and collide. The attracting state is all defectors, and it is reached after 1007 games. Figure 9 shows the spatial pattern following game 301. The yellow players are the advancing defectors, and the green players are the advancing cooperators.	1%	Clusters of defectors grow around 4 initial defectors. The results of game 3 are shown, and the 4 clusters of defectors are still discernible. These clusters of defectors combine and continue growing until a steady-state solution with 96.25% defectors is reached after 12 games. In this particular case, the steady-state solution consists of a single 3×5 cluster of cooperators.
10%	The attracting state is reached after 14 games. Defectors form in "lines" partitioning the space. Along the lines, localized 2 and 3-period oscillations result in a global 6-period response.	10% (a)	Again, the percentage of defectors grows quickly in the first few games. The regions of cooperators and defectors each expand, contract and collide. The attracting state is all defectors, and is reached after 255 games. Figure 9 shows the spatial pattern following game 217, when the percentage of defectors reaches a minimum of 15.5%.	10% (a)	Regions of defectors expand about the initial defectors, until reaching a steady-state solution (shown) with 89.5% defectors. The attracting state is a single 6×7 cluster of cooperators, which is reached after the 6th game.

50% (a)	The attracting state consists of all defectors, and is reached after 2 games. The results of game 1 are shown. Initial defectors are in red; yellow players turned defector after 1 game; blue players are cooperators. In this case, the two remaining cooperators will defect after the next game, while the rest of the players will continue to defect.	10% (b)	The percentage of defectors fluctuates "chaotically" about an average of 67.1% until the 212th game., when a 9-period solution forms as the attracting state. The 2 symmetric regions of cooperators shown in Fig. 9 expand and then contract, forming the 9-period solution.	10% (b)	Regions of defectors expand about the initial defectors, until reaching a steady-state solution (shown) with 88.5% defectors. The attracting state is reached after the 5th game, and consists of 3 clusters of cooperators. The cluster sizes are 3×3, 4×3, and 5×5.
50% (b)	The attracting state is reached after 2 games, and consists of a traveling wave. The traveling wave moves from right to left through the lattice. The wave pattern is 2-period as players alternate states; the global pattern is 20 period as the wave circles the 20×20 lattice of the torus.	10% (c)	The percentage of defectors fluctuates about an average of 69.41%, until a 100-period solution forms after game 1196. The 100-period solution has large variations about an average of 50.60% defectors.	10% (c)	Regions of defectors expand about the initial defectors, until reaching a steady-state solution (shown) with 84.0% defectors. The attracting state is reached after the 8th game, and consists of 3 clusters of cooperators. The cluster sizes are 3×3, 5×5, and 6×5.
50% (c)	After the first game, the lattice consists of 98% *defectors*. A 2×2 cluster of cooperators begins expanding, resulting in a 2-period solution after 24 games. In this case, the players shown in green alternate between cooperating (green) and defecting (yellow).	50%	The attracting state (shown) is 100% defectors, and is reached after just 2 games.	50%	The attracting state (shown) is 100% defectors, and is reached after just 1 game.

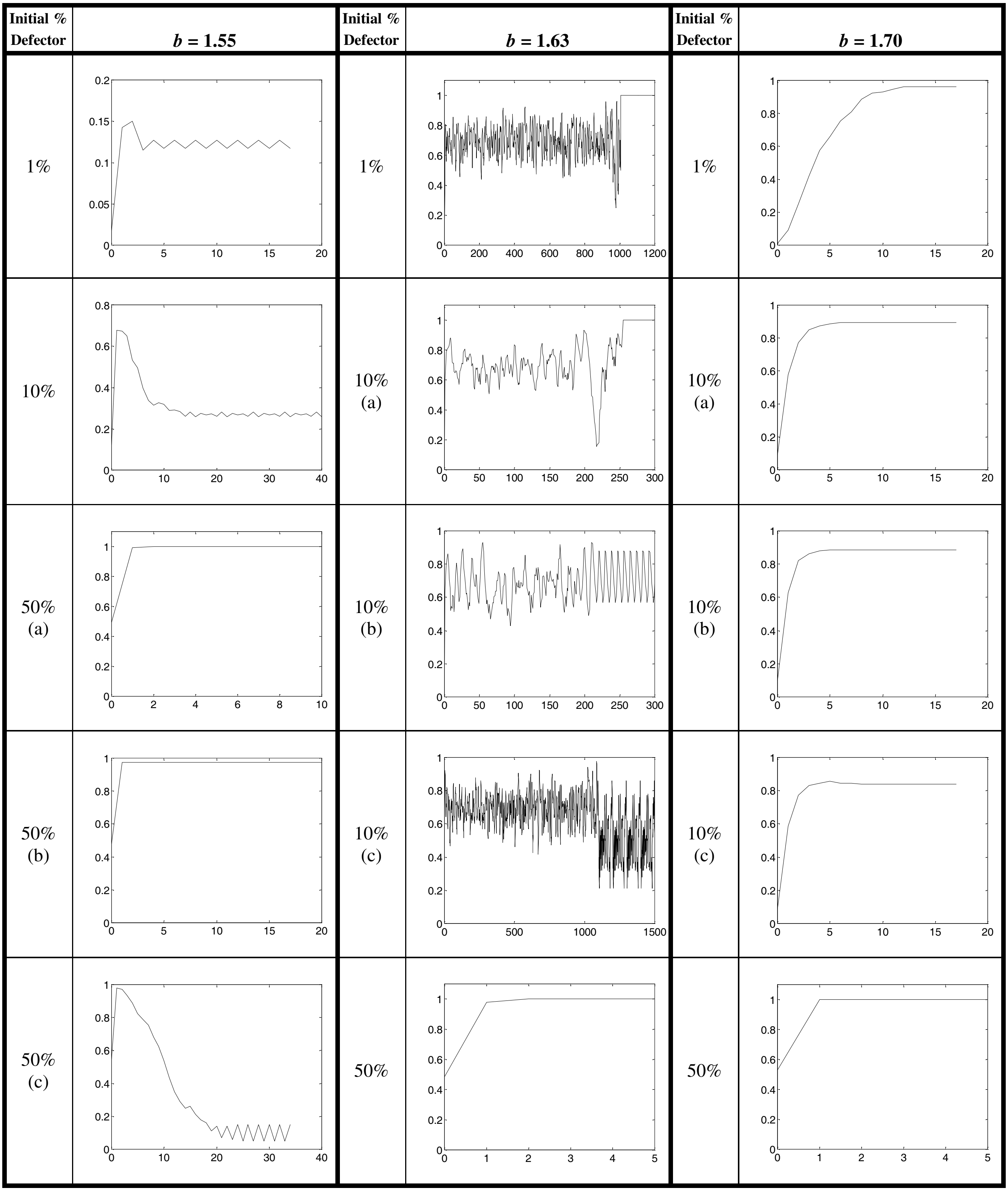


Fig. 8. The percentage of defectors versus the number of completed games for $b = 1.55$, 1.63 and 1.70 for various random initial distributions of defectors. The results are shown for a 20×20 lattice with periodic boundary conditions and without self-interaction, and correspond to the results in Table 4 and Fig. 9. Similar results seem to occur regardless of lattice size. (Note that the vertical and horizontal scales change from chart to chart to optimize the clarity of the information provided.)

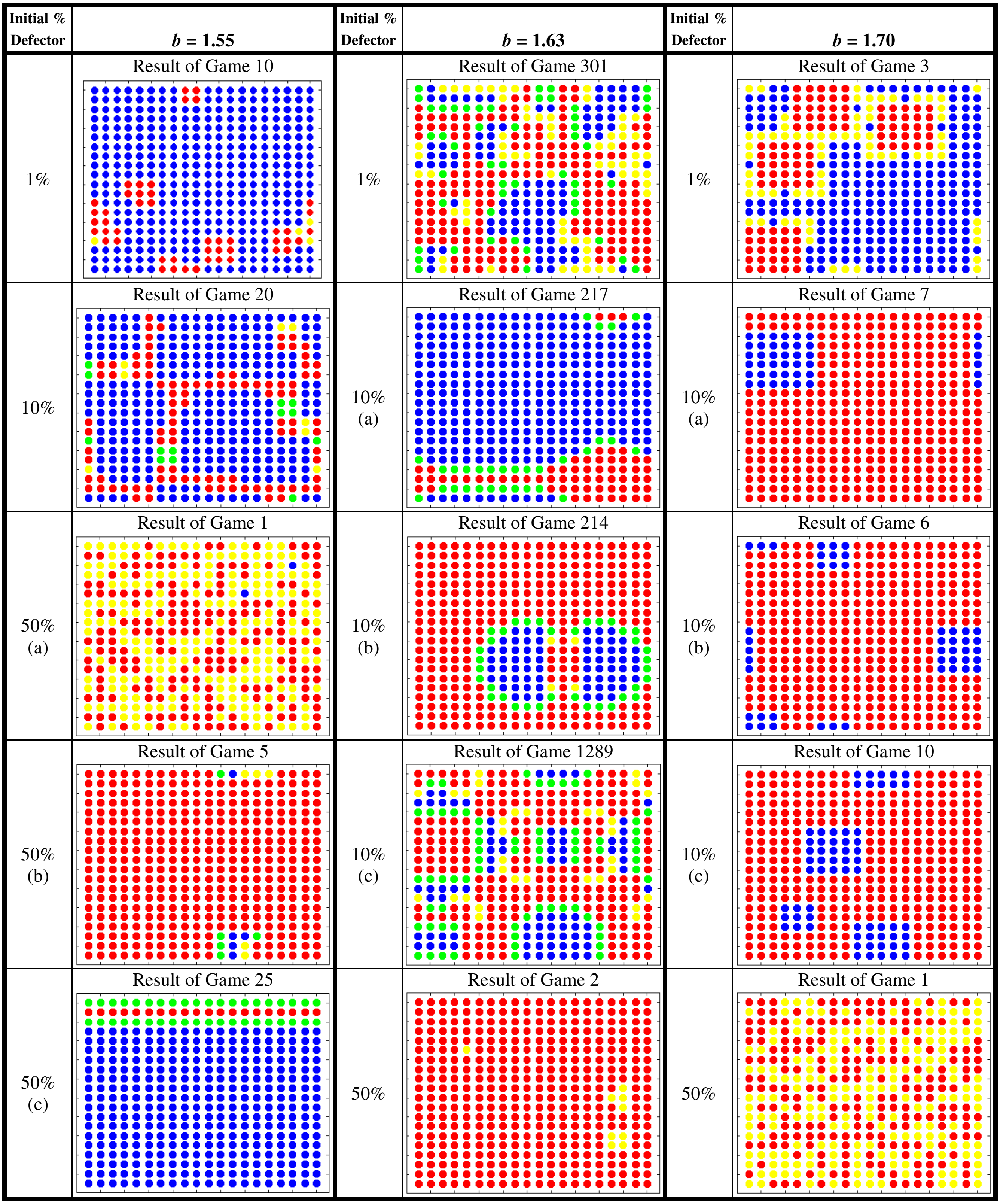


Fig. 9. Representative spatial patterns for $b = 1.55$, 1.63 and 1.70 for various random initial distributions of defectors. The results are shown for a 20×20 lattice with periodic boundary conditions and without self-interaction, and correspond to the results in Table 4 and Fig. 8. Green cells show the advancement of cooperators (blue), and yellow cells show the advancement of defectors (red). For variety, some of the spatial patterns presented are the steady-state attracting state, some are a part of a periodic solution, and some are representative of the dynamics (see Table 4 for a detailed description).

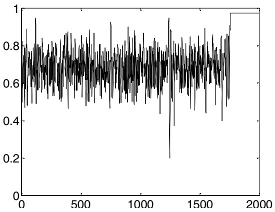
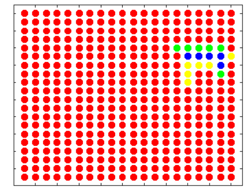
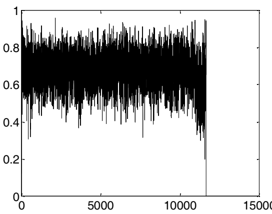
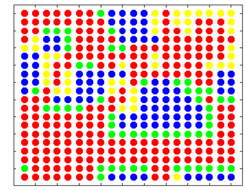
Initial % Defector	Description	Percentage of Defectors vs Game	Representative Spatial Pattern
10% (a)	The attracting state is reached after 1757 games, and consists of a traveling wave. The traveling wave moves vertically upward through the lattice. The wave pattern is 2-period as players alternated states; the global pattern is 20 period as the wave circles the entire torus. The attracting state has a constant 97.25% defectors, even though the wave pattern is 2-period.		Result of Game 1760 
10% (b)	The spatial pattern appears to chaotically fluctuate with an average of 68.62% defectors, and then suddenly, and unexpectedly, converges to an attracting state of all cooperators (0% defectors) after 11,633 games.		Result of Game 993 

Fig. 10. Typical results with $b = 1.85$ for a 20×20 lattice with periodic boundary conditions, *with* self-interaction, and with a 10% initial distribution of defectors are shown. When self-interaction is included, this highly dynamic region occurs for $9/5 < b < 2$.

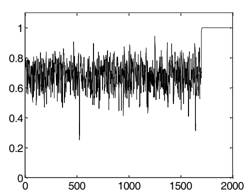
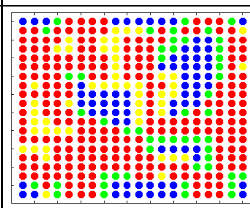
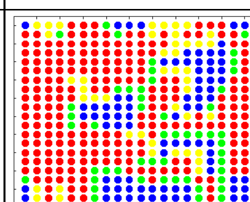
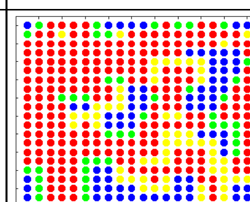
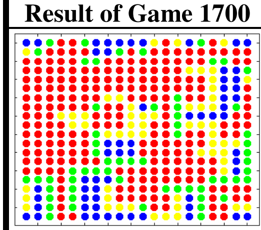
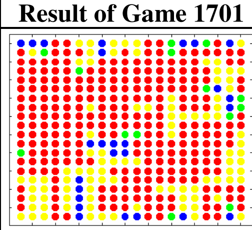
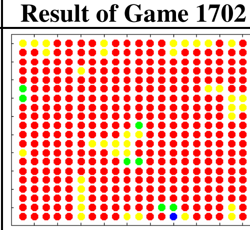
Description	Percentage of Defectors	Result of Game 1697	Result of Game 1698	Result of Game 1699
Spatial patterns appear to chaotically fluctuate, with an average of 68.03% defectors, until suddenly and unexpectedly, they converge to the attractive state of 100% defectors after 1703 games.				
Result of Game 1700	**Result of Game 1701**	**Result of Game 1702**	**Result of Game 1703**	**Result of Game 1704**
				

Fig. 11. The spatial development for a 20×20 lattice, with periodic boundary conditions and *with* self-interaction is shown for $b = 1.85$ and a 10% random initial distribution of defectors. In this particular case, the attracting state is 100% defectors and is reached suddenly after 1703 games. The evolution of several games is shown to give a sense of the complicated dynamics that occur.

$b = 1.55$, $b = 1.63$ and $b = 1.70$ to represent the three different regions. Descriptions of typical results for different initial percentages of defectors are given in Table 4, and graphically displayed in Figs. 8 and 9. Though most of the results presented below are for a 20 by 20 lattice, similar results are generally obtained for 29 by 29 and 59 by 59 lattices.

With $b = 1.55$, a 1% random initial distribution of defectors results in attracting states that include local steady-state and periodic solutions in the regions surrounding the initial defectors. A 5% and a 10% random initial distribution of defectors results in "lines of defectors" partitioning the lattice. Along these lines, or boundaries, local steady-state and periodic solutions exist. Observed periodicities include 2, 3, 4 and 6-period solutions at various locations along the lines of defectors. The global periods are multiples of the local periods. While the attracting states for 1%, 5% and 10% initial distribution of defectors are somewhat

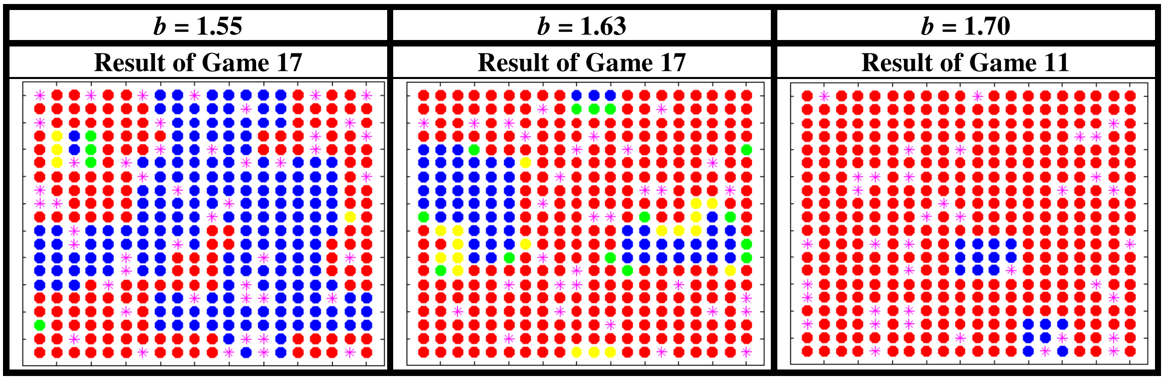


Fig. 12. Typical results for a 20×20 lattice, with periodic boundary conditions and without self-interaction, are shown for $b = 1.55$, 1.63 and 1.70. The initial condition is a 10% random distribution of defectors, and a 10% random distribution of ideal cooperators. The color scheme is the same as used previously, except that the ideal cooperators are shown as magenta colored asterisks. The attracting state is shown for $b = 1.55$, and contains three different 2-period solutions. It has 40.25% defectors, and is reached after the 14th game. The dynamic state after game 17 is shown for $b = 1.63$. The ideal cooperators limit the growth of the clusters of cooperators. This attracting state is reached after the 23rd game and consists of all defectors except for the ideal cooperators and a small, localized 2-period solution. The attracting state is shown for $b = 1.70$. It is reached after the 11th game and is all defectors, except for a 3×4 and a 3×3 cluster of cooperators and the ideal cooperators.

predictable, the attracting states for a 50% random initial distribution of defectors are not. The sensitivity to initial conditions now becomes strongly manifest. For, while starting with a 50% random initial distribution of defectors, numerous different periodic solutions asymptotically result for different initial distributions of defectors. Three very different attracting states for a 50% initial distribution of defectors are illustrated in Table 4, and shown in Figs. 8 and 9. These include an attracting state that is 100% defectors and is reached after just two games, see (a) in Table 4; an attracting state that is a "traveling wave" that moves from right to left throughout the lattice, see (b) in Table 4; and an attracting state that is a 2-period solution that forms a straight line through the lattice, see (c) in Table 4. Notice the wide difference in the percentage of cooperators in the attracting states (a) and (c) as seen in Figs. 8 and 9.

Similar results are obtained for a b value of 1.45 using a lattice size of 29 by 29. Again, sensitivity of the global pattern to local patterns of distribution of defectors is observed. Depending on the nature of the "local" distribution of cooperators in small neighborhoods of the lattice, within a few rounds of play, a self-organization into small regions (nuclei) of cooperators can result; these nuclei then expand, collide, and along their lines of collision usually generate defectors that then asymptotically result in lines, blocks and islands of defectors. Depending on the number and location of nuclei generated, numerous different asymptotic states can result. Where the local distribution is not conducive to the creation of these nuclei of cooperators, steady-state (1-period) asymptotic solutions are often generated.

Sensitive dependence on the "local" initial distribution of cooperators also shows its influence on lattice size. Increasing the lattice size results in a greater propensity for periodic solutions, there being greater opportunities for local distributions to engender nuclei of cooperators which then expand, collide, etc. Simulations for a lattice size of 59 by 59 confirm these observations. Also, though the nature of the asymptotic states differs widely, they are achieved relatively rapidly, in a few 10's of games.

Based on these results and also our simulations with 10% to 90% of defectors randomly placed initially on larger lattice sizes of 29 by 29 and 59 by 59, the dynamical behavior observed in this regime has three basic characteristics: (1) the percentage of cooperators can fluctuate widely *during* the evolution of the dynamics; (2) numerous different periodic solutions with lines and islands of defectors separating zones of cooperators can asymptotically result; (3) there is very sensitive dependence on initial conditions, leading to wide diversity of asymptotic states — that range from all cooperators to all defectors — for any given percentage of the initial random distribution of defectors.

The unpredictability of the final asymptotic state, and sensitivity to initial conditions, is highlighted by the fact that with an initial random distribution of defectors of as high as 90%, asymptotic states with *all* cooperators might still be generated.

With $b = 1.63$, the attracting state is generally either steady-state or periodic, but the number of games required to reach the attracting state varies significantly, for any given initial percentage of defectors. Typical examples are described in Table 4, and shown in Figs. 8 and 9. In the given example for a 1% random initial distribution of defectors, the spatial pattern ultimately reaches an attracting state with 100% defectors, but it takes 1,007 games to get there. Periodic solutions are also possible within this region of b values.

Three different representative results are shown for a 10% initial distribution of defectors with $b = 1.63$. One possible attracting state is 100% defectors. For the case shown, 100% defectors is reached after 255 games, see (a) in Table 4. A second attracting state consists of a 9-period solution with expanding and contracting regions of cooperators. The dynamics appears to be chaotic until the 9-period solution is suddenly formed, see (b) in Table 4. A third attracting state consists of a 100-period solution that is formed after 1,196 games. This particular case averaged 69.4% defectors through the game prior to the formation of the 100-period solution, and 50.6% defectors after the formation of the 100-period solution, see (c) in Table 4. Other results include attracting states with different periods, and attracting states with 100% cooperators.

This wide variety of steady-state and/or periodic attracting states appears to occur independent of lattice size, and this was verified for lattices with dimensions between $n = 9$ and $n = 29$. In general, the number of games required to reach the attracting state increases as the size of the lattice increases, and we presume that similar results will generally occur for much larger lattices, and that a very large number of games may be required to reach the attracting states. For example, for a 10% random initial distribution of defectors, an increase in the lattice size from 20 by 20 to 29 by 29 causes the number of games needed to reach asymptotic behavior to increase from several hundred to several thousands (typically 25–30,000). For a lattice size of 20 by 20, a 50% initial distribution of defectors seems to consistently result in 100% defectors after just a few games. Larger lattice sizes lead to a greater variety of asymptotic solutions, like traveling waves, even when the initial distribution of defectors exceeds 50%.

With $b = 1.70$, the attracting state is similar to the attracting states observed for simple, symmetrical initial conditions. With a 1% initial random distribution of defectors, the clusters of defectors grow and combine, until a steady-state attracting state with a high percentage of defectors is reached. The steady-state solution includes rectangular clusters of cooperators. Similar results are observed with a 10% initial distribution of defectors, as indicated by the three different examples shown in Table 4, and Figs. 8 and 9. The differences in the attracting states are simply the size and number of the rectangular clusters of cooperators (even for lattices sizes of 29 by 29 and 59 by 59). A 50% initial distribution of defectors typically results in an attracting state with 100% defectors.

Perhaps the most significant finding when looking at the effect of random initial distributions of defectors on the system dynamics is the extreme fluctuations in the percentages of defector populations as the dynamics evolve, especially for values of b between 8/5 and 5/3. We see that during the evolution of the dynamics, population sizes of 98% defectors can dramatically reduce to populations of about 10% defectors in the steady state; and, populations that may have as low as 15% defectors at some stage in the evolution of the dynamics can suddenly explode to have near 100% defectors — all this while the rules of the game played by each player remain unchanged!

(f) Consideration of self-interaction, solely for comparison purposes, when starting from random initial distributions of defectors.

The results so far obtained have been for the situation where a player does not play against himself/herself, that is to say, when there is no self-interaction. For comparison with results that might arise when self-interaction is included, we present in this section some results with self-interaction. Thus, the player is essentially allowed to compete against himself, and hence adds the gains of his own self-interaction in order to obtain his cumulative score in each round, before he decides on his strategy for the next round.

Perhaps the most pronounced effect of allowing self-interaction is the shift it causes in the highly dynamic transitional region (region F, Table 2) from the range of b values from $8/5 < b < 5/3$ to the range $9/5 < b < 2$. Within this region, attracting states with 100% defectors, 100% cooperators, traveling waves, and various periodicities were observed. Square lattices with dimensions $n = 9$, 15, 20, 24, and 29 are studied. Typical results for a 20×20 lattice with $b = 1.85$ are shown in Figs. 10 and 11. Note that after what looks like a random fluctuation in the percentage of defectors, the system enters, quite abruptly, a basin of attraction. In addition to the bifurcation values in the gain b presented in Table 1, when self-interaction is included bifurcation values may include $b = 9/8$, $9/7$, $9/6$, $9/5$, $9/4$, $9/3$, $9/2$, $9/1$.

(g) Influence of including ideal cooperators — individuals who will not defect no matter what their gains — on the global patterns.

Going back to the context of criminal behavior that we broached in our introductory section, we examine the effect of including players who are ideal cooperators. An ideal cooperator is a player who will never change to being a defector, no matter what his/her cumulative gains are compared to his/her neighbors, and therefore, regardless of whether his/her most-profitable neighbors are defectors. The ideal cooperators have the same payoff scheme as a normal cooperator. The ideal cooperator therefore never changes state, and no players are allowed to become ideal cooperators after the game is initiated.

A 20 by 20 lattice with periodic boundary conditions and without self-interaction is studied. The initial condition consists of approximately 10% defectors randomly distributed across the population. Further, 10% of the players are taken to be "ideal cooperators" and they too are randomly distributed in the population. We again consider the regions: $b < 8/5$; $8/5 < b < 5/3$; and, $b > 5/3$. Typical results with no self-interaction and with $b = 1.55$, 1.63 and 1.70 are shown in Fig. 12. The presence of ideal cooperators can change the evolutionary dynamics substantially: it can greatly affect the transient dynamics and reduce the number of games needed to reach an attracting state; and, it can totally change the qualitative nature of the asymptotic state. Furthermore, ironically, the inclusion of ideal cooperators tends to *increase* the percentage of defectors in the attracting state for $b < 8/5$. Similar results are obtained on 29 by 29 lattices.

For $b = 1.55$, the connecting "lines of defectors" observed for the same conditions without the "ideal cooperators" largely disappear (compare Figs. 9 and 12). Periodic solutions still occur, and the asymptotic state is typically reached in a smaller number of games. Figure 13 shows a typical result when varying the percentage of ideal-cooperators, while always starting with a randomly distributed, 90% cooperator (and 10% defector) population. For comparison purposes, all the simulations are conducted using the same random seed so that the random placement of initial defectors is identical in all cases. As seen from the figure, the asymptotic state with no ideal cooperators is periodic and the percentage of defectors is about 30%; as the percentage of ideal-cooperators increases, the defector population in the asymptotic state increases. Note also the rapid convergence to the asymptotic behavior caused by the constraint imposed by the presence of the ideal cooperators. Even when 70% of the cooperators are ideal cooperators, the asymptotic percentage of defectors exceeds that which arises when no ideal cooperators exist! This is because defectors are "attracted" around the ideal

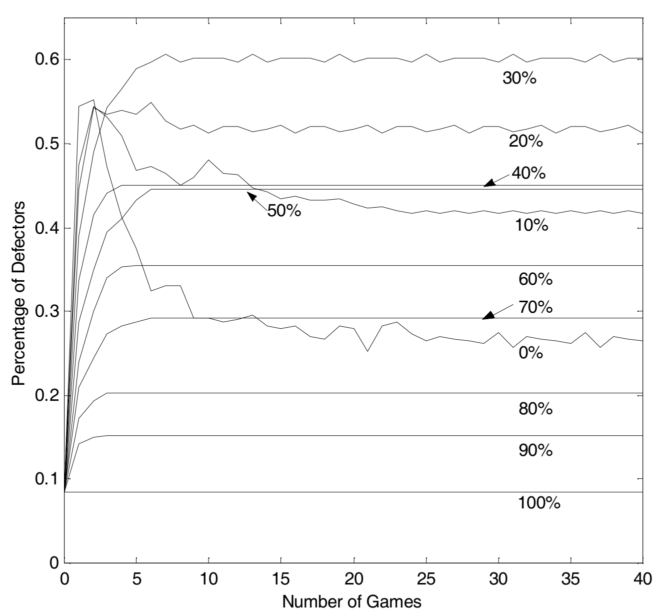


Fig. 13. This graph demonstrates that including ideal cooperators, ironically, tends to increase the percentage of defectors. The percentage of defectors versus the number of games completed is plotted for different percentages of ideal cooperators. For comparison, the exact same 10% random distribution of initial defectors is used throughout, and the percentage of initial cooperators that are "ideal" cooperators is increased from 0% to 100%. The percentage of ideal cooperators is annotated on each curve. This evaluation was conducted for a 20×20 lattice with periodic boundary conditions and without self-interaction, for $b = 1.55$.

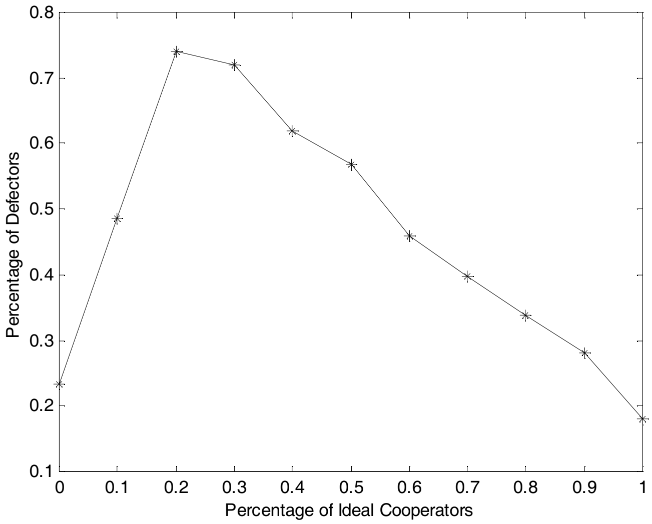


Fig. 14. The percentage of defectors (at the end of 50 games) versus the percentage of ideal cooperators is plotted for the case of 80% total initial cooperators (normal cooperators plus ideal cooperators). For comparison, the same distribution of initial cooperators is used, as the percentage of ideal cooperators is increased. The presence of ideal cooperators appears to increase the percentage of defectors in the asymptotic state. This evaluation was conducted for a 20×20 lattice with periodic boundary conditions and without self-interaction, for $b = 1.55$.

cooperators, for they continue to gain from their unwavering behavior.

Figure 14 shows similar behavior when starting with an 80% population of cooperators, which is uniformly distributed across the lattice. The percentage of ideal cooperators is varied, as before, in this cooperator population. The plot shows the percentage of defectors (at the end of 50 games) versus the percentage of ideal cooperators among the initial 80% of the cooperator population. Again, the presence of ideal cooperators appears to increase the percentage of defectors in the asymptotic state.

For $b = 1.63$, the dynamics are substantially more constrained than they are without the "ideal cooperators" (compare Figs. 9 and 12). Defectors again fill in around the ideal cooperators and tend to form boundaries that limit the growth of regions of cooperators. The attracting states are reached in a substantially smaller number of games with ideal cooperators than without them. Fluctuations in the percentage of defectors during the transient dynamics are much attenuated. Steady-state and periodic solutions still occur. The inclusion of ideal cooperators can still cause the number of defectors in the asymptotic state to increase as compared to when they are not present.

When $b = 1.70$, the results are largely unaffected by the addition of ideal cooperators (compare Figs. 9 and 12). The attracting state tends to be a steady-state solution with a few rectangular clusters of cooperators that may also contain the ideal-cooperators.

(h) Discussion on the use of periodicities and percentages of defectors as metrics for understanding the dynamics.

The periodicities and percentage of defectors in the attracting state of each region are reasonable indicators of the dynamics that take place in each region. However, there are many subtleties that these parameters mask. A few of these subtleties are mentioned here:

(1) For the initial condition DCCCCCD and $b < 8/5$, the periodicities were the same as the initial case of a single defector (D). A spacing of five cooperators between defectors fully isolates the defectors from each other. The center cooperator is essentially unaware that the defectors exists. The specific periodic solutions is the same as those shown in Fig. 2 for this range of b values, except they occur separately around each defector.
(2) For the initial condition DCCCD, there are two distinct 3-period solutions adjacent to each other in the parameter ranges $6/5 < b < 4/3$ and $4/3 < b < 7/5$.
(3) In some cases, the percentage of defectors is constant, although the actual spatial pattern contains periodic solutions.
(4) The global periodicity may be due to the superposition of local periodic solutions of equal or lesser periodicities. For example, local 2-period and 3-period solutions can result in a global 6-period solution.
(5) An n-period behavior in a graph of percentage of defectors versus number of the games completed does not necessarily imply an n-periodic dynamical state. The periodicity of the dynamical state may be much higher because of the fact that it is related to the *pattern* of defectors and not just their numbers.

3. Conclusions

In this paper we investigate in some detail the emergence of global patterns from local interactions that arise in the iterated Prisoner's Dilemma game with *no* self-interaction.

Making a very simple assumption on the nature of local interactions, i.e. that each player will follow the lead of the most "successful" player in his/her immediate neighborhood, produces surprisingly complex global patterns with complicated dynamics and sometimes unpredictable results. We observe steady-state solutions, simple and complex periodic solutions and traveling waves. None of the simulations we conducted in this study (with, and without, self-interaction) gave rise to solutions that were truly chaotic, nor did they give rise to states that seemed to fluctuate unpredictably without end.

The level of detail in our investigation in this paper appears to be greater in many respects than that available hereto. This has led to a deeper understanding of the interaction dynamics: our ability to categorize the patterns into three principal b-regions (we do not simply show various patterns of interaction, as has been the usual practice so far); the sensitivity of the patterns to perturbations in the initial conditions; the presence of "perturbulence" type phenomena similar to that found in other large-scale coupled dynamical systems [Udwadia & von Bremen, 2002]; the striking lack of long term chaotic behavior; and the physical explanation for the counterintuitive behavior when ideal cooperators are included.

Several specific regions for the defector's payoff, b, exist with predictable bifurcation values. While the detailed attracting states and dynamics within each specific region of b values varies, three general regions appear to dominate.

For $b < 8/5$, the primary players affected appear to be largely in the local neighborhood of the initial defectors. The region of defectors grows further only when the clusters around the initial defectors are themselves in contact with each other.

A transitional region exists for payoff values with $8/5 < b < 5/3$. This appeared to be a sort of "marginally stable" region, similar to perturbulence, with relatively long transients, and attracting states that range from 100% defectors to 100% cooperators, and include steady-state and long period solutions. Results are extremely sensitive to the initial conditions. Attracting states are reached suddenly, and without warning, sometimes after numerous iterations of the game.

For $b > 5/3$, the attracting state is a steady-state with a high percentage of defectors. When cooperators exist in the attracting state, they are localized in small, sort-of rectangular clusters. The presence of ideal cooperators tends to constrain the dynamic expansion of clusters of cooperators when $b < 5/3$. For $b > 5/3$, ideal cooperators tend to have little effect on the results.

It is interesting to note, as shown in Fig. 4, that the percentage of defectors in the final steady-state solution that is generated by a single defector in a field of cooperators depends not only on the defector's gain b, but also on the size of the lattice (the number of players in the community). Also noteworthy, as shown in Fig. 8, are the extreme fluctuations in the percentage of defectors that can arise as the dynamical system evolves, and the precipitous manner in which a seemingly chaotic fluctuation in the percentage of defectors gets attracted to a steady-state solution.

In general, increasing the lattice size appears to increase the number of games needed to reach asymptotic behavior, sometimes very substantially. Also, the number of qualitatively different asymptotic states for a given initial percentage of defectors (when starting with a random distribution of defectors) appears to increase with lattice size.

The inclusion of ideal-cooperators — individuals who refuse to defect no matter what their gains — leads to some surprising results. They could influence both the transient dynamics and the qualitative nature of the attracting state. Depending on the region in which b lies, their inclusion could *increase* the number of defectors in the asymptotic state compared to when these ideal cooperators are absent. Their inclusion generally reduces the number of games needed to reach the asymptotic state, as well as the fluctuations in the percentage of defectors during the evolution of the transient dynamics.

We have focused on the dynamics and attracting states of the spatial patterns for different initial conditions and payoffs. Our aim is to see if such simple models might exhibit characteristics that could throw some light on the evolution of social, political and economic development and their patterns. We notice that even with this simple multi-person dynamical system, the outcome, i.e. the global patterns generated, are often very complex, alter drastically as the system dynamically evolves, and often defy prediction. Thus very complex global behavior can be engendered by simple local rules of interaction.

Our model of interaction has four major limitations: first, we assume that all the participants play each round in a synchronous manner; second, we assume that each person's wealth, and the total wealth of the "closed" community (in which each individual is assumed to be statically located on the torus), is not constrained in any way; third, that each player plays only against his nearest neighbors, and hence the players have no spatial mobility; and fourth, that each player is privy to complete and accurate information about the gains and behavior of his/her neighbors. Real-life situations of interaction usually are far more complex, and one may presume that, in general, they could lead to even greater complexities of dynamical behavior than those observed here.

Even our simplistic model indicates that the global patterns generated can be so complex that it may be difficult to find useful, simple-minded explanations for real-life phenomena like "crime waves" (wide dynamic fluctuations in crime statistics), and oscillations in the stock market. Therefore, these results indicate the challenges in making simple-minded predictions of the global patterns of social and economic phenomena by pointing out that even if they are dependent on just a few, simple, deterministic rules of local interaction, their behavior is complex and very sensitive to initial conditions. We note that such predictions appear difficult to make not because of any inherent uncertainties, or reasons like bounded rationality of the interacting agents; they arise because of the inherent nonlinearity — albeit simple to characterize — in the local interactions among the agents.

Despite the challenges, it seems plausible that with appropriate assumptions on local interactions, and proper characterization of the lattice and the initial conditions, this multidimensional approach could lead to useful modeling and simulation of certain social, political and economic phenomena. The complexity of the dynamics suggests that, in general, the best way to determine the global dynamical evolution for different assumptions on local behavior, is to "run the simulation." Given the scarcity of mathematical tools available today for handling such systems, computer simulations appear to be the most reasonable approach to understanding and predicting their qualitative global behavior.

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