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Distributed Control of Large-Scale Structural Systems

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Abstract: *This article develops a methodology for the distributed active control of large structural systems. The central idea is to use a set of active members in a structural system, each* locally *controlled. The method relies on an understanding of* (1) *the peculiar interaction of force feedback and velocity feedback in an active member and* (2) *the interaction between such active members, as well as that between active and passive members in a structural system. Using a commonly occurring MDOF model of a structural system, we show that the distributed-local-control design is robust. Locating the active members so that all the eigenvalues of the damping and stiffness matrices of the controlled system are assuredly increased leads to several results on where and how active members might be located in such a structure.*

Controlling large-scale structures in dynamic loading environments so that they fulfill the needs for which they are built is a challenge to both the structural dynamics community and the controls community. As such, the task is a difficult one. The purposes for structural control are varied and depend on the goals or objectives for which the structural system is designed. For example, in precision structures that are deployed for the purpose of making accurate measurements from orbiting satellites, the aim might be to ensure that the relative displacements and distortions in the structure are sufficiently small to maintain certain "lines of sight" to within prescribed tolerances. In the control of structures such as dams and tall buildings that may be subjected to strong earthquake ground shaking, the aim of structural control is to maintain the response of the structure to within safe and acceptable limits in terms of both the stresses induced and the story drifts.

Depending on the goals to be achieved, different control methodologies have been investigated by various researchers. Yet there are certain underlying features that are common to most situations where structural control is employed. Any successful methodology has to contend with them. They are

1. Uncertainty in the structural model
2. Uncertainty in the nature of the dynamic loading environment
3. Economics and reliability of the control system
4. Power requirements and the response time of the control system
5. Ability to operate successfully (or at least nondetrimentally) under conditions of complete or partial sensor and/or actuator failure
6. Possible instabilities caused by the noncollocation of sensors and actuators in large structural systems
7. Actuator dynamics and feasibility issues

In addition, for large structures such as dams, bridges, and buildings, massive control forces may need to be generated to fulfill the purposes of structural control, thus calling for multiple actuators and thus the problem of their possible interaction and possible instabilities. Precision shape control of large structures such as antennas is another area where interaction between actuators may be significant.

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The approach offered in this article attempts to provide a control methodology that is robust in the sense of the issues just mentioned and which, we believe, is feasible in terms of today's technology. We show that through the use of active structural members in a structural system, i.e., members that can actively generate forces between their points of connection, we can effectively control the response of structural systems while still being cognizant of the issues mentioned earlier. The methodology proposed is that of distributing such members throughout the structure to both reduce its global response to vibrations and dampen them out rapidly, each active member being *locally* controlled through the measurement of the force acting through it and the relative motion of its ends.

The use of control systems that employ displacement feedback and velocity feedback has been well researched in the past. This includes the use of proof masses and pulse control. It originated in the control community and has been employed, with some degree of success, in certain structural systems. Yet it leaves several of the issues that we have stated earlier unattended or only partially so at best. The following summary of some references supports our argument. In this article, we propose the combination of force feedback along with velocity feedback. Such a feedback methodology was first proposed by G.-S. Chen and B. Lurie,[4] who used a bridge feedback concept to feed back a combination of signals from sensors of the axial force and relative velocity across the active member. Experimental results using multiple active members in a truss structure can be seen in refs. 2 and 3. Another technique that has been investigated is called *positive-position feedback control*, and this makes use of generalized displacement measurements to accomplish vibration suppression using piezoelectric materials for actuators and sensors.[6] Development of active members using piezoelectric and electrostrictive actuation has been worked out by Anderson, Moore, and Fanson.[1] The problem of optimal placement of active/passive members in large-scale structures is considered in refs. 4 and 5. These authors have attempted to solve the problem on the basis of engineering judgment, in which they have adopted the maximization of the cumulative energy dissipated over a finite time interval as the measure of optimality. An interesting approach to increase the low inherent damping of large-scale structures is given in ref. 7. These authors have considered two types of passive and active joints and have shown that these joints are able to give the structure higher levels of passive damping without significantly increasing the structure's weight or complexity. In this article, by analyzing the stability of the entire structure in the presence of the active members, we show the robustness of the distributed-local-control methodology and how the positive force feedback leverages the damping. These have not been looked at by other researchers and make our approach significantly different from others.

Our methodology relies on two things: (1) the simultaneous use of several active members distributed throughout the structure and (2) the peculiar interaction between force feedback and velocity feedback in every active member. It is these two elements that provide a viable approach that ensures robustness and high reliability while using local control.

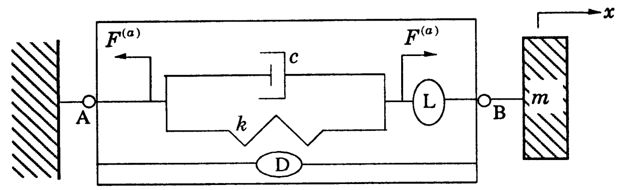


Fig. 1. Mechanical model of active member AB.

This article is divided into two sections. The first deals with the mechanical modeling of an active member. The second deals with the use of such members in a simple structure modeled as an MDOF system to show the interaction between different active members and between active members and passive members; here, we present several stability results related to the distributed-local-control methodology that is proposed in this article.

1 THE ACTIVE MEMBER

Consider a structural element that is active in the sense that it can generate a force between the two points to which it is connected. Such members have been developed and used recently in structural systems, though so far in only a limited, experimental capacity. A description of an as-built active member using piezoelectric wafers to create the internally generated active force is provided in Appendix 1.

A simple mechanical model of such a member supporting a mass is shown in Figure 1. The active member AB is capable of generating an internal force $F^{(a)}$ between its two ends that can be controlled. We assume that the internal force $F^{(a)}$ can be generated instantaneously, and for this study, we ignore the detailed actuator dynamics. When this force equals zero for all time, the member becomes a passive member with stiffness k and viscous damping c. The load cell L measures the force transmitted to the mass. The relative displacement between the two ends of the active member is measured by the displacement cell D and later differentiated to provide the relative velocity between the ends A and B of the active member.

The mass m may be thought of as composed of the external mass that the active member is connected to, as well as a contribution from the member's own lumped mass, as is commonly done in structural modeling. The equation of motion of the mass m is then simply

$$m\ddot{x} + c\dot{x} + kx = F^{(a)} \tag{1}$$

If the feedback force $F^{(a)}$ is made a function of (1) the force f_L measured by the load cell L, (2) the relative velocity $\dot{x}$ between the two ends of the active member, and (3) the relative displacement x between its ends, then we have the relation

$$F^{(a)} = f_1(f_L) + f_2(\dot{x}) + f_3(x) \tag{2}$$

where the force f_L measured by the load cell L is given by

$$f_L = F^{(a)} - kx - c\dot{x} \tag{3}$$

and the functions $f_1(\cdot)$, $f_2(\cdot)$, and $f_3(\cdot)$ are as yet unspecified.

A simple feedback relation would have these functions simply as multiplicative constants so that $f_1(f_L) = g(F^{(a)} - kx - c\dot{x})$, $f_2(\dot{x}) = -r\dot{x}$, and $f_3(x) = -sx$. The constants g, r, and s are then the force-feedback gain, the velocity-feedback gain, and the displacement-feedback gain, respectively. Using these relations in Eq. (2), we get

$$F^{(a)} = -\frac{g}{1-g}(kx + c\dot{x}) - \frac{r}{1-g}\dot{x} - \frac{s}{1-g}x \tag{4}$$

Equation(1) now becomes

$$m\ddot{x} + c^{(a)}\dot{x} + k^{(a)}x = 0 \tag{5}$$

where the effective stiffness k_a and the effective damping c_a of the active member AB are given by

$$k^{(a)} = \frac{k}{(1-g)} + \frac{s}{(1-g)} \tag{6}$$

and

$$c^{(a)} = \frac{c}{(1-g)} + \frac{r}{(1-g)} \tag{7}$$

We note that though we are actively controlling the system, the active member is "fooled" into thinking that it is a passive member with an effective stiffness k_a and an effective damping c_a! Behaving as though it were passive has obvious advantages from a control stability viewpoint, especially when dealing with MDOF systems containing several active members.

The feedback thus causes the effective undamped natural frequency of the active member to be altered so that

$$\omega_n^{(a)} = \frac{\omega_n}{\sqrt{(1-g)}}\sqrt{1 + \frac{s}{m\omega_n^2}} \tag{8}$$

and its effective percentage of critical damping to be altered so that

$$\zeta_n^{(a)} = \frac{\zeta_n}{\sqrt{(1-g)}} \frac{[1 + (r/2m\omega_n\zeta_n)]}{\sqrt{1 + (s/m\omega_n^2)}} \tag{9}$$

where $\omega_n = \sqrt{k/m}$ and $\zeta_n = c/2m\omega_n$ are the characteristics of the member when passive.

When $g < 1$, the effective stiffness constant of the member is reduced ($k^{(a)} < k$) if $s < -kg$; also, the effective damping constant of the member is reduced ($c^{(a)} < c$) if $r < -gc$. Usually the gains r and s are taken to be positive, thereby providing both negative velocity and negative displacement feedback.

In particular, Eq. (6) indicates that with no displacement feedback ($s = 0$), the effective stiffness is always reduced when the force-feedback gain g is negative. The active member appears "softer," and the displacement response typically increases. Similarly, with no velocity feedback ($r = 0$), the effective damping in the system reduces ($c^{(a)} < c$) when the force-feedback gain g is negative.

On the other hand, when $s > -kg$, the effective stiffness constant $k^{(a)}$ is increased, and when $r > -gc$, the effective damping constant $c^{(a)}$ is also increased. With negative displacement feedback ($s \geq 0$), a positive force-feedback gain, $0 < g < 1$, always causes the right-hand side of Eq. (6) to increase. Also, for a positive force-feedback gain, $0 < g < 1$, the first term on the right-hand side of Eq. (7) increases, leveraging the inherent damping of the passive member by the factor $1/(1-g)$. Similarly, if r is positive (i.e., we are using negative velocity feedback), then a positive force-feedback gain $g < 1$ again leverages the effect of the velocity feedback by the factor $1/(1-g)$, thereby causing the second term on the right-hand side to be larger than r. Thus the damping of the active member is increased dramatically due to increases in both the terms on the right-hand side of Eq. (7). The increased effective stiffness of the member in this situation results in a "stiffer" member, and the displacement response typically reduces.

To illustrate the effect that this leverage has on changing the damping characteristics of the active member, we consider a system whose passive characteristics are as follows: mass m equal to unity, natural period equal to unity, and percentage of critical damping equal to $\frac{1}{2}$%. We consider no displacement feedback, i.e., $s = 0$. Figure 2 shows the locations of the poles of the transfer function corresponding to Eq. (5) as the force-feedback gain g is varied from 0.8 through 0 to -1 and the velocity-feedback gain r is varied from 0 to unity in steps of 0.2 units. The figure shows five curves each for a different value of r. The right-most curve corresponds to the value of $r = 0$; each succeeding curve going toward the left corresponds to a value of r that is 0.2 units greater than the preceding one. The plus signs mark the locations of the roots for $g = 0.8$, the small oh's mark the locations where $g = 0$, and the asterisks mark the locations where $g = -0.8$. Observe that along any one of the lines, the poles on the plot do not change their locations very much when g changes from 0 to -0.8; the damping remains essentially unaltered, while the stiffness decreases. As g is gradually increased from zero toward 0.8, the roots change substantially, due to the leveraged interaction between the force feedback and the velocity feedback, shifting the poles leftward and dramatically increasing the damping in the system.

Figure 3 shows how the percentage of critical damping

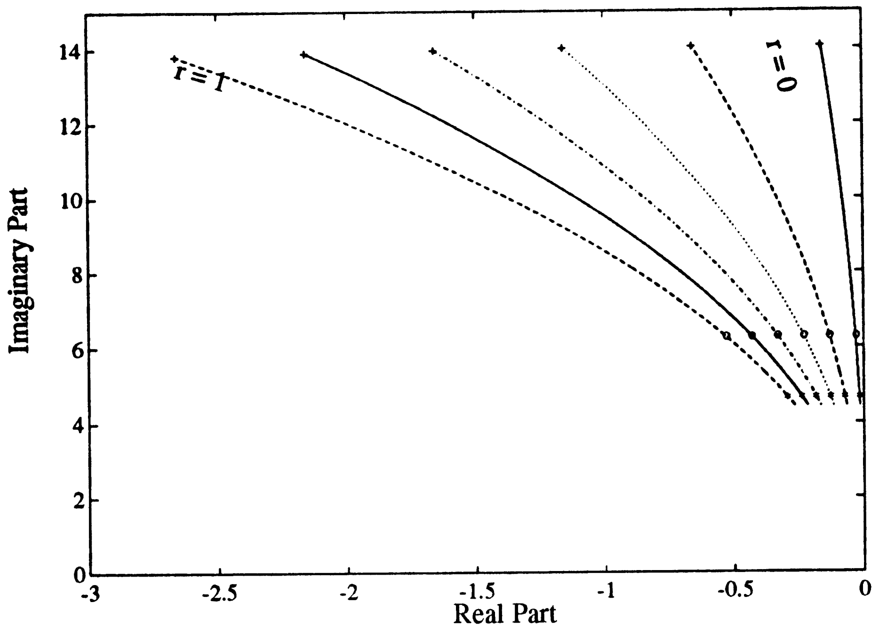


Fig. 2. The poles of the system described by Eq. (5) as g and r are varied.

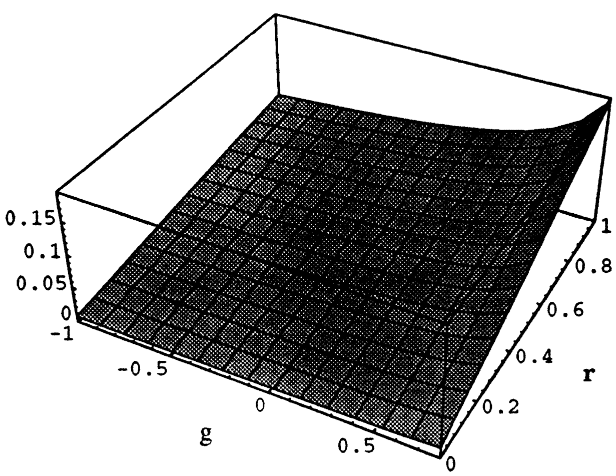


Fig. 3. The interaction between velocity feedback and force feedback.

$\zeta_n^{(a)}$ changes with the parameters g and r. We observe the interaction between the velocity feedback (r ranging from 0 to 1) and the force feedback (g ranging from -1 to 0.8). For positive values of g, the percentage of critical damping $\zeta_n^{(a)}$ is increased substantially by the presence of the velocity feedback. The figure shows how positive force feedback leverages the damping. The percentage of critical damping has increased from $\frac{1}{2}\%$ to 20%.

Figure 4 shows the displacement response of the mass m when attached to the active member and subjected to an impulsive force of 5 units. The passive characteristics of the member are $m = 1$, natural period = 1, and percentage of critical damping = 1%. We compare here the effects of positive force feedback, with $g = 0.75$, and an equal negative force feedback, with $g = -0.75$, keeping the velocity feedback a constant, with $r = 0.5$. The solid line represents the response of the passive member, the dashed line represents the response of the active member with positive force feedback, and the dot-dash line represents the response of the active member with negative force feedback. As expected, positive force feedback stiffens the active member, reduces the amplitude of motion, and because of the increased damping caused by the interaction between the force and velocity feedback, brings the system to rest rapidly. The frequency of the response increases when compared with the response of the passive member. The negative force feedback causes increased displacements and shows a softening of the member. The response decays slowly when compared with that obtained using positive force feedback.

Figure 5a shows the response of the same system to the uniformly distributed random base acceleration shown in Figure 5b. The initial displacement and velocity of the system are taken to be zero. We again compare the effectiveness of positive force feedback (with $g = 0.75$) and negative force feedback (with $g = -0.75$), keeping the velocity-feedback gain fixed with $r = 0.5$.

The response of the passive member is shown by the solid line, the response of the active member with positive force feedback is shown by the dashed line, and the response with the negative force feedback is shown by the dot-dash line.

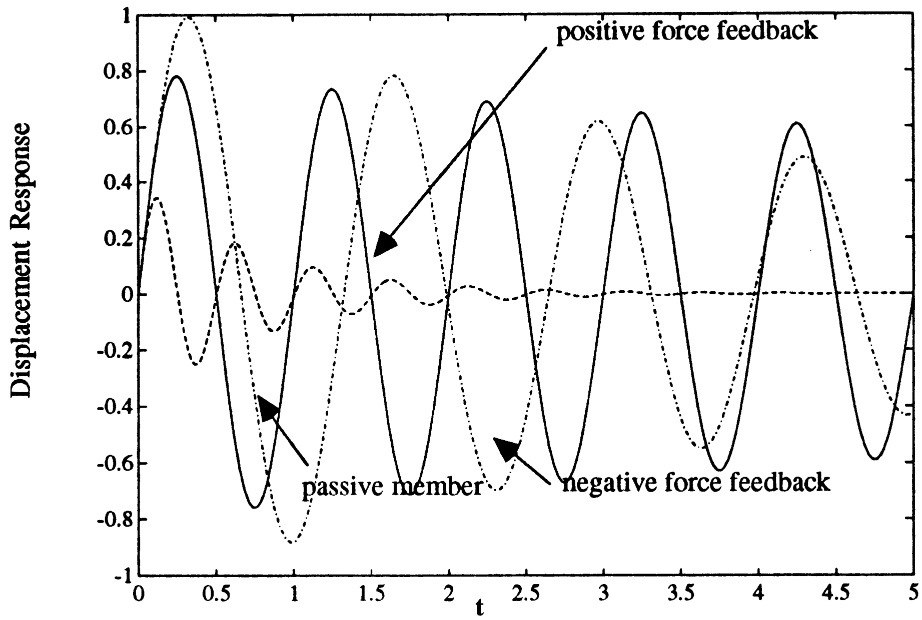


Fig. 4. The difference in impulse response between positive force feedback and negative force feedback to the active member.

The response with positive force feedback is substantially reduced compared with that using negative force feedback. The structure is stiffened by positive force feedback and softened by negative force feedback.

2 INTERACTION BETWEEN ACTIVE MEMBERS

We consider here the interaction that may result between several active members placed in a structure. We consider a simple structure that might represent the model of a tall building subjected to strong earthquake ground shaking. Furthermore, such a model, intuitively speaking, promises a rich interaction between the active and passive members as well as interactions among the active members themselves.

2.1 The structural model

Figure 6 shows an MDOF model that utilizes active members. The passive stiffness and damping constants for the ith member are k_i and c_i, respectively, for $i = 1, 2, \ldots, n$. The n lumped masses are m_i, $i - 1, 2, \ldots, n$, as shown. The motion of the n degree of freedom system is described by the equation

$$M\ddot{\mathbf{x}} + C\dot{\mathbf{x}} + K\mathbf{x} = \mathbf{F}^{(a)}(t) - M\mathbf{i}\ddot{u}(t) + \mathbf{F}^{(e)}(t) \qquad (10)$$

where $\ddot{u}(t)$ is the base acceleration, $\mathbf{x}$ is the n vector of motion relative to the base, and the n vector $\mathbf{i} = [1\ 1\ 1\ \cdots\ 1]^T$. The n by n matrix $M = \mathrm{Diag}\{m_1, m_2, \ldots, m_n\}$. The external forces applied to the system (if there are any) are denoted by vector $\mathbf{F}^{(e)}(t)$.

The n vector $\mathbf{F}^{(a)}(t)$ on the right-hand side of Eq. (10) is a consequence of the fact that we are using active members. Its ith component can be expressed as

$$\mathbf{F}_i^{(a)}(t) = F_i(t) - F_{i+1}(t) \qquad 1 \le i \le n \qquad (11)$$

where we define $F_{n+1}(t) = 0$. The active force created by the ith active member is denoted here as $F_i(t)$. The matrices K and C are each symmetric and tridiagonal, with elements

$$K_{i,i} = k_i + k_{i+1} \qquad C_{i,i} = c_i + c_{i+1} \qquad 1 \le i \le n \quad (12)$$

and

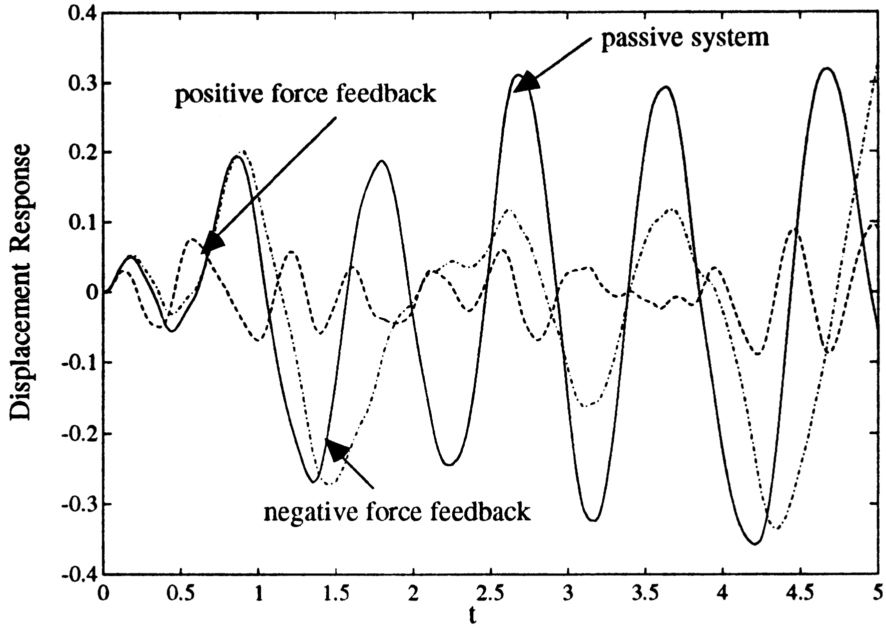
$$K_{i,i+1} = -k_{i+1} \qquad C_{i,i+1} = -c_{i+1} \qquad 1 \le i \le n-1 \quad (13)$$

where we define $k_{n+1} = c_{n+1} = 0$.

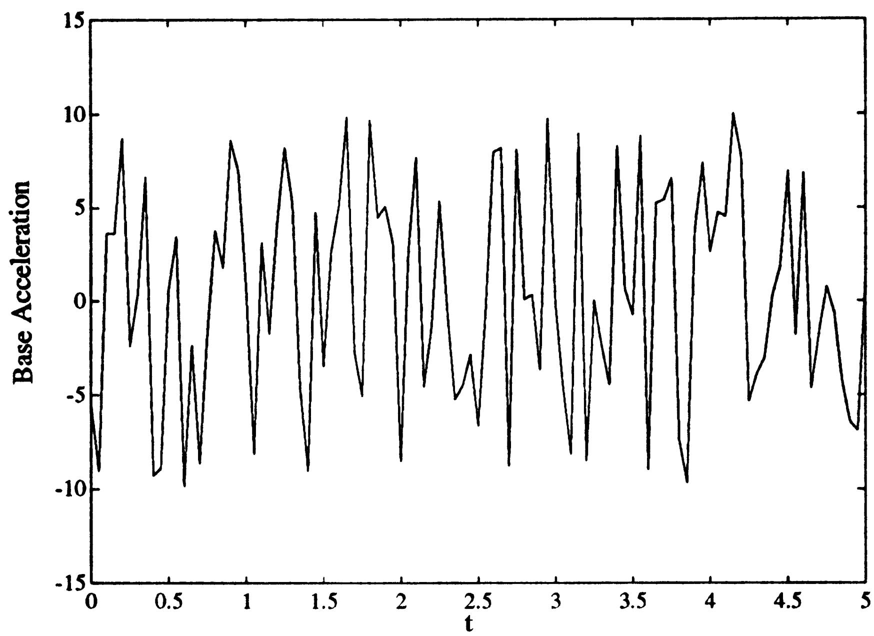
The force F_i in the ith active member can be written in an analogous fashion to the preceding section as

$$\begin{aligned} F_i &= g_i[F_i - k_i(x_i - x_{i-1}) \\ &\quad -c_i(\dot{x}_i - \dot{x}_{i-1})] - r_i(\dot{x}_i - \dot{x}_{i-1}) \\ &\quad -s_i(x_i - x_{i-1}) \end{aligned} \qquad (14)$$

for $1 \le i \le n$ (with $x_0 = \dot{x}_0 = 0$). The constants g_i, r_i, and s_i in Eq. (14) are, respectively, the force-feedback gain, the velocity-feedback gain, and the displacement-feedback gain corresponding to the ith active member. Hence the force F_i generated by the active member in view of the feedback



(a)



(b)

Fig. 5. (a) Response to random excitation. (b) Base acceleration.

relation [Eq. (14)] becomes

$$F_i = -\frac{g_i}{(1-g_i)}[k_i(x_i - x_{i-1}) + c_i(\dot{x}_i - \dot{x}_{i-1})] - \frac{r_i}{1-g_i}(\dot{x}_i - \dot{x}_{i-1}) - \frac{s_i}{1-g_i}(x_i - x_{i-1}) \quad (15)$$

for $1 \leq i \leq n$.

In what follows, unless explicitly stated, we will assume that the active members are used with positive force feedback so that $0 \leq g_i \leq 1, i = 1, 2, \ldots, n$, and negative velocity and displacement feedback so that $r_i, s_i, \geq 0, i = 1, 2, \ldots, n$.

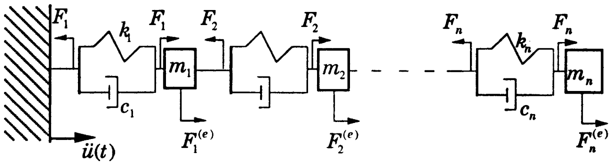


Fig. 6. The structural model of the MDOF system with active members.

Using Eq. (15) in Eq. (11) to determine the n vector $\mathbf{F}^{(a)}(t)$, Eq. (10) now becomes

$$M\ddot{\mathbf{x}} + C^{(a)}\dot{\mathbf{x}} + K^{(a)}\mathbf{x} = -M\mathbf{i}\ddot{u}(t) + \mathbf{F}^{(e)}(t) \tag{16}$$

The elements of the symmetric tridiagonal matrices $K^{(a)}$ and $C^{(a)}$ are given by

$$\begin{aligned} K^{(a)}_{i,i} &= \frac{k_i + s_i}{1 - g_i} + \frac{k_{i+1} + s_{i+1}}{1 - g_{i+1}} \\ C^{(a)}_{i,i} &= \frac{c_i + r_i}{1 - g_i} + \frac{c_{i+1} + r_{i+1}}{1 - g_{i+1}} \\ & \quad 1 \le i \le n \end{aligned} \tag{17}$$

and

$$\begin{aligned} K^{(a)}_{i,i+1} &= -\frac{k_{i+1} + s_{i+1}}{1 - g_{i+1}} \\ C^{(a)}_{i,i+1} &= -\frac{c_{i+1} + r_{i+1}}{1 - g_{i+1}} \\ & \quad 1 \le i \le n-1 \end{aligned} \tag{18}$$

where in Eq. (17) we define $g_{n+1} = r_{n+1} = s_{n+1} = 0$.

We are now ready to look at the interaction between the active members and between the active members and passive members. We observe that to convert the ith active member to a passive member, all we need do, then, is set the gains g_i, r_i, and s_i to zero.

2.2 Stability of the distributed-local-control methodology

We begin by investigating whether the system composed of active and passive members, where one or more of the active members may be located at any of the n possible locations, can become unstable because of the local feedback control that is used in each active element. We note that the feedback force in any active member is only cognizant of the *local* variables related to that particular member, i.e., the force measured by the load cell in *that* active member and the relative displacement and velocity between *its* two ends. Thus the control is of a distributed *local* nature throughout the structure. Investigation of the stability of the entire system under the influence of these *local controllers* is therefore necessary to establish if our methodology is to be viable. We will assume that $0 \le g_i < 1, \forall i \in$ any subset of $\{1, 2, \ldots, n\}$ and that $r_i, s_i \ge 0, \forall i \in$ any subset of $\{1, 2, \ldots, n\}$.

Remark 1: The local control described above, with positive force feedback $0 \le g_i < 1, \forall i \in$ any subset of $\{1, 2, \ldots, n\}$, and negative velocity and negative displacement feedback $r_i, s_i \ge 0, \forall i \in$ any subset of $\{1, 2, \ldots, n\}$, cannot destabilize the system, provided that the passive system is stable. By *passive system*, we mean the corresponding system whose members are all passive. In this remark we will not include the possibility of the first member being an active member. (This case is taken up in Remark 2 and yields a stronger result.)

We prove this by showing that the stiffness matrix of the controlled structure $K^{(a)}$ is always such that $K^{(a)} \ge K$ and the damping matrix $C^{(a)} \ge C$. We consider first that only the ith member ($i > 1$) in the structure is active, the other members being all passive. Then it is easy to see that

$$K^{(a)} = K + \Delta K \tag{19}$$

where the matrix ΔK has zeros everywhere except for the 2 by 2 symmetric block whose elements are

$$\Delta K_{i-1,i-1} = \Delta K_{i,i} = \frac{g_i k_i + s_i}{1 - g_i} \tag{20}$$

and

$$\Delta K_{i-1,i} = \Delta K_{i,i-1} = -\frac{g_i k_i + s_i}{1 - g_i} \tag{21}$$

This 2 by 2 block has rank 1, and its eigenvalues are zero and $[2(g_i k_i + s_i)]/(1 - g_i)$. The latter eigenvalue is positive as long as $s_i > -g_i k_i$, a condition that is satisfied. Thus the matrix $\Delta K \ge 0$; i.e., it is positive semidefinite. If we were to arrange the eigenvalues of K and $K^{(a)}$ in ascending order, then the eigenvalues of the active system are greater than or equal to the corresponding eigenvalues of the passive system.

Similarly, the elements of the matrix $\Delta C = C^{(a)} - C$ are all zero except for the 2 by 2 symmetric block, whose elements are

$$\Delta C_{i-1,i-1} = \Delta C_{i,i} = \frac{g_i c_i + r_i}{1 - g_i} \tag{22}$$

and

$$\Delta C_{i-1,i} = \Delta C_{i,i-1} = -\frac{g_i c_i + r_i}{1 - g_i} \tag{23}$$

The eigenvalues of this 2 by 2 block are again zero and $[2(g_i c_i + r_i)]/(1 - g_i)$. The latter eigenvalue is positive because $r_i > 0$ and $0 < g_i < 1$. Hence the matrix ΔC is positive semidefinite. Taking $E(t) = \frac{1}{2}(x^T K^{(a)} x + \dot{x}^T M \dot{x})$ as the energy of the active system when subjected only to an initial disturbance [i.e., with $\mathbf{F}^{(e)} = \mathbf{0}$, and $\ddot{u}(t) = 0$], we find that $\dot{E}(t) = -\dot{x}^T C^{(a)} \dot{x} \le -\dot{x}^T C \dot{x}$, and hence the active system is stable if the passive system is stable.

Consider next two active members that are not adjacent to each other. We exclude the first element being an active member (this case is taken up later). The elements of ΔC

will be all zero now except for two disjoint 2 by 2 symmetric blocks each of a form similar to those described in Eqs. (22) and (23). Our results would again hold, the proof being similar. The same reasoning can be extended to several active members no two of which are adjacent to each other.

Now consider the case where two active members are adjacent to each other; say, the ith and $(i+1)$st members are active. Then the elements of ΔC will all be zero except for a 3 by 3 symmetric block that is positive semidefinite. Again ΔC will be positive semidefinite, and the result follows.

Remark 2: The eigenvalues $\lambda_i(C^{(a)}) > \lambda_i(C)$ whenever the member closest to the base of the structure is active. Similarly, $\lambda_i(K^{(a)}) > \lambda_i(K)$, and the structure is globally stiffened by the presence of this active member.

For the member closest to the base, $i = 1$. To prove this result, we first note that the eigenvalues of the matrix $C^{(a)}$ are continuously dependent on the parameters that describe the elements of the matrix. Furthermore, by looking at a perturbation study of the eigenvalues of $C^{(a)} = C + \Delta C$, we show in Appendix 2(a) that for small perturbations,

$$\lambda_p(C^{(a)}) \approx \lambda_p(C) + \alpha_p \mathbf{x}_p^T \Delta C \mathbf{x}_p \tag{24}$$

where $\lambda_p(C)$ is the pth eigenvalue of C, $\mathbf{x}_p$ is the corresponding eigenvector of C, and α_p is a positive number.

When the only active element in the system is the first number, the elements of ΔC are all zero except the (1, 1) element, which is $(g_1c_1 + r_1)/(1 - g_1)$. Equation (24) then implies that

$$\lambda_p(C^{(a)}) \approx \lambda_p(C) + \alpha_p \frac{g_1c_1 + r_1}{1 - g_1}(x_p^1)^2 \tag{25}$$

where x_p^1 is the first element of the pth eigenvector of the matrix C. In Appendix 2(b), however, we show that $x_p^1 \neq 0$ for all p. Hence every eigenvalue of $C^{(a)}$ is greater than the corresponding eigenvalue of C, so ΔC is strictly positive definite. Starting from some initial state, the rate of energy dissipation therefore assuredly increases compared with the passive system.

A similar argument follows for the matrix $K^{(a)}$. The structure is "globally" stiffened, and all the "natural" frequencies of the system [Eq. (16)] are higher than those of the system [Eq. (10)].

Remark 3: If a set of active members is contiguous with the first member ($i = 1$), which is also active, then $\lambda_i(C^{(a)}) > \lambda_i(C)$, $\lambda_i(K^{(a)}) > \lambda_i(K)$, for all i.

Say the first q members are all active. Then the matrix ΔC has elements that are zero except for the first q by q symmetric tridiagonal block. This block is positive definite. Using Eq. (24), we find that

$$\lambda_p(C^{(a)}) \approx \lambda_p(C) + \alpha_p \mathbf{x}_p^T \Delta C \mathbf{x}_p > \lambda_p(C) \qquad \text{for all } p \tag{26}$$

and hence our result is proved, noting that the first q elements of any eigenvector $\mathbf{x}_p$ of C cannot all be zero. A similar proof follows for the matrix $K^{(a)}$. Hence in this case too the structure is globally stiffened, and the rate of energy dissipation is increased.

Remark 4: If two (or more) active members are contiguously placed, then, again, the matrices $C^{(a)}$ and $K^{(a)}$ are such that $\lambda_i(C^{(a)}) > \lambda_i(C)$, $\lambda_i(K^{(a)}) > \lambda_i(K)$, for all i.

Let us say that the ith and $(i+1)$st elements are active. Then the elements of ΔC will all be zero except the 3 by 3 symmetric tridiagonal block given by

$$\Delta C_{i-1,i-1} = \frac{g_ic_i + r_i}{1 - g_i} \qquad \Delta C_{i-1,i} = -\frac{g_ic_i + r_i}{1 - g_i} \tag{27}$$

$$\Delta C_{i,i} = \frac{g_ic_i + r_i}{1 - g_i} + \frac{g_{i+1}c_{i+1} + r_{i+1}}{1 - g_{i+1}}$$

$$\Delta C_{i,i+1} = -\frac{g_{i+1}c_{i+1} + r_{i+1}}{1 - g_{i+1}} \tag{28}$$

and

$$\Delta C_{i+1,i+1} = \frac{g_{i+1}c_{i+1} + r_{i+1}}{1 - g_{i+1}} \tag{29}$$

This block has rank 2 and is positive semidefinite, and the eigenvector corresponding to the zero eigenvalue of this block is $[1\ \ 1\ \ 1]^T$. Now Eq. (24) yields

$$\lambda_p(C^{(a)}) \approx \lambda_p(C) + \alpha_p \mathbf{x}_p^T \Delta C \mathbf{x}_p \tag{30}$$

But the second term on the right must be positive, for it could only be zero if three consecutive components of the eigenvector are each unity, which by Appendix 2(c) is impossible. A similar argument holds for the matrix $K^{(a)}$. Hence the result.

Remark 5: If the nth element is an active member, then $\lambda_i(C^{(a)}) > \lambda_i(C)$, $\lambda_i(K^{(a)}) > \lambda_i(K)$, for all i, resulting in a global stiffening of the structure.

Here the elements of ΔC will all be zero except for the 2 by 2 symmetric block given by

$$\Delta C_{n-1,n-1} = \Delta C_{n,n} = \frac{g_nc_n + r_n}{1 - g_n} \tag{31}$$

and

$$\Delta C_{n-1,n} = \Delta C_{n,n-1} = -\frac{g_nc_n + r_n}{1 - g_n} \tag{32}$$

Again, we note that the 2 by 2 symmetric block is positive semidefinite and that the zero eigenvalue for this block results only for the eigenvector $[1\ \ 1]^T$. However,

$$\lambda_p(C^{(a)}) \approx \lambda_p(C) + \alpha_p \mathbf{x}_p^T \Delta C \mathbf{x}_p \tag{33}$$

and the second term on the right can only be zero if the $(n-1)$th and nth elements of the eigenvector $\mathbf{x}_p$ are identical. Appendix 2(d) shows that this can never happen. Hence

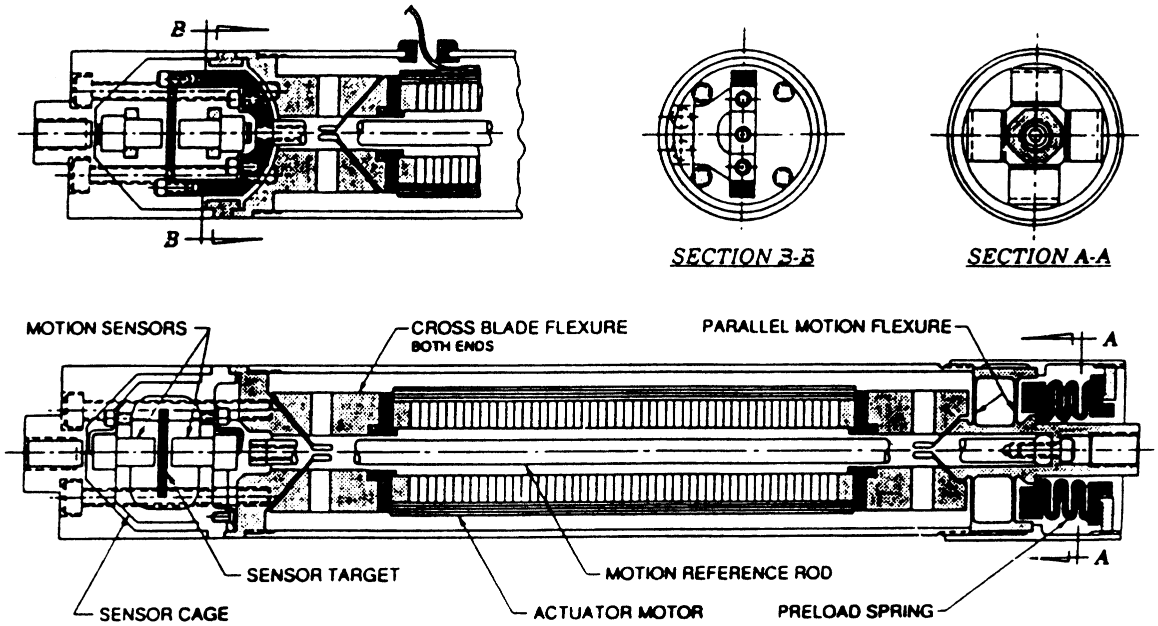


Figure A.1. Drawing of the active member.

the second term on the right must be positive, and hence $\lambda_p(C^{(a)}) > \lambda_p(C)$ for all p. The result for $K^{(a)}$ can be proved similarly.

Remark 6: If a set of active members is contiguous with the last member ($i = n$), which is also an active member, then $\lambda_i(C^{(a)}) > \lambda_i(C)$, $\lambda_i(K^{(a)}) > \lambda_i(K)$, for all i. This is obvious using Eq. (33) and noting the result of Appendix 2(c).

3 CONCLUSIONS

In this article we have considered the use of active members for the distributed local control of large-scale structures. We have shown that some of the currently available active members can be modeled in a simple manner in which the force feedback interacts with the velocity feedback. This interaction can be used to good advantage through the use of positive force feedback, which leverages not only the velocity feedback but also the inherent damping of the passive member. We show that the active member can be stiffened and its damping can be increased dramatically using this approach.

We next use such active elements locally controlled, and distributed throughout an MDOF system. Such MDOF systems commonly occur in many areas of structural dynamics. We assess the global stability of the MDOF system under the type of distributed localized control that we consider in this article. We conclude that such distributed local control cannot destabilize the system.

The active control alters the stiffness and damping matrices of the structure from its passive values of K and C, to $K^{(a)}$ and $C^{(a)}$. All the eigenvalues of the matrices $K^{(a)}$ and $C^{(a)}$ are increased (when compared with those of K and C, respectively) through the use of positive force feedback and negative velocity feedback, resulting in a global stiffening of the structure and, generally speaking, an increased rate of energy dissipation when

1. The member connecting the system to the base of support is active
2. Two or more active members are placed contiguously anywhere in the system
3. Active members are placed contiguously with the member connecting the system to its base of support or they are placed contiguously with the member farthest from the base of support.

When a single isolated active member is placed in the system, unless it is the member closest to or farthest away from the base of support, the global stiffness is not assuredly increased, nor are the eigenvalues of the damping matrix assuredly increased.

These results may have considerable significance in the control of large structures whose models are uncertain and which operate in uncertain dynamic loading environments. The methodology is robust with respect to actuator power failures, for should one or more of the active members no longer remain active, the structure is not destabilized. We do away with the necessity of a centralized control unit that could then be susceptible to failure. The application of the force being collocated with the sensor leads to no spillover effects. The interaction between the active members and between the active and passive members can be designed so as to greatly increase the damping in the system without any detrimental side effects. Procedures for doing this, and ways of choosing the various parameters that describe the active and the passive members, will be taken up in subsequent work.

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APPENDIX 1

Figure A.1 shows a drawing of one active member[1] built to replace members of truss structures for its control. The specifications for the member are

Overall length: < 8 inches (200 mm)
Nominal diameter: 1.0 inch (25 mm)
Zero displacement force: > 100 pounds (450 N)
Zero force displacement: ± 1 mil ($\pm 25\mu$m)
Displacement sensing range: ± 4 mils ($\pm 100\mu$m)
Displacement sensing resolution: 1 μin (25 nm)
Stiction: (zero)

The piezoelectric stacks are built up of annular wafers with 15 mm outer diameter and 8 mm inner diameter. The thickness of each wafer of 1 mm, requiring a maximum operating voltage of 1000 V with a bias of 500 V. The mechanical preload in the wafers retains the piezoelectric material in compression during its contraction motion. The active member provides relative displacement information and actuation. The stroke is adjustable by controlling the length of the piezoelectric stack (almost 0.1% maximum strain from the piezoelectric material), and the diameter establishes the force capability.

APPENDIX 2

(a) Let C be an n by n symmetric matrix with eigenvalues λ_i, $i = 1, 2, \ldots, n$, and corresponding orthogonal eigenvectors x_i, $i = 1, 2, \ldots, n$. Consider the eigenvalue problem

$$(C + \Delta C)(\mathbf{x}_i + \Delta\mathbf{x}_i) = (\lambda_i + \Delta\lambda_i)(\mathbf{x}_i + \Delta\mathbf{x}_i) \qquad \text{(A1)}$$

where we have the perturbation ΔC added to the matrix C, causing the new eigenvalues to be $(\lambda_i + \Delta\lambda_i)$ and the new eigenvectors to be $\mathbf{x}_i + \Delta\mathbf{x}_i$. Noting that $C\mathbf{x}_i = \lambda_i\mathbf{x}_i$, and ignoring terms of second order, we then obtain

$$(C - \lambda_i I)\Delta\mathbf{x}_i = (\Delta\lambda_i I - \Delta C)\mathbf{x}_i \qquad \text{(A2)}$$

Let the vector $\mathbf{y}_i = \sum_{j=1}^n b_j\mathbf{x}_j = (\Delta\lambda_i I - \Delta C)\mathbf{x}_i$ and $\Delta\mathbf{x}_i = \sum_{j=1}^n a_j\mathbf{x}_j$ because the orthogonal eigenvectors $\mathbf{x}_i$, $i = 1, 2, \ldots, n$, span the space R^n. Using these relations in Eq. (A2), we get

$$\sum_{j=1}^{n} a_j(C - \lambda_i I)\mathbf{x}_j = \sum_{j=1}^{n} b_j\mathbf{x}_j \qquad \text{(A3)}$$

which implies that

$$a_j(\lambda_j - \lambda_i) = b_j \qquad j = 1, 2, \ldots, n \qquad \text{(A4)}$$

For the equation set (A4) to be consistent, we require, then, that $b_i = 0$, and hence y_i must be orthogonal to $\mathbf{x}_i$. Thus

$$\mathbf{x}_i^T\mathbf{y}_i = \mathbf{x}_i^T(\Delta\lambda_i I - \Delta C)\mathbf{x}_i = 0 \qquad \text{(A5)}$$

which yields

$$\Delta\lambda_i = \alpha_i\mathbf{x}_i^T\Delta C\mathbf{x}_i \qquad \text{(A6)}$$

where $\alpha_i = (\mathbf{x}_i^T\mathbf{x}_i)^{-1}$ so that

$$\lambda_i(C+\Delta C) = \lambda_i(C) + \alpha_i(\mathbf{x}_i^T\Delta C\mathbf{x}_i) \quad i = 1, 2, \ldots, n \quad \text{(A7)}$$

(b) Since C (and $C^{(a)}$) is a symmetric tridiagonal matrix and has the structure described in Eqs. (12) and (13), if the first element of any eigenvector is zero, then the entire vector must be zero.

(c) Consider the eigenvectors of C, where C has the structure described in Eqs. (12) and (13). The matrix is positive definite, and hence all the eigenvalues are positive. If an eigenvector $\mathbf{x}$ exists such that its ith, $(i-1)$th, and $(i+1)$th components are identical (say, normalized to unity), then the ith equation of the set $C\mathbf{x} = \lambda\mathbf{x}$ would be

$$- c_{i-i,i} + (c_{i,i} + c_{i,i+i}) - c_{i,i+1} = 0 = \lambda \tag{A8}$$

implying that an eigenvalue of C is zero, therefore leading to a contradiction. Hence no eigenvalue of C can have an eigenvector with three consecutive components equal.

(d) Suppose that the $(n-1)$th and nth components of an eigenvector of C are identical (say, normalized to unity). As in part (c) above, the last equation of the set $C\mathbf{x} = \lambda\mathbf{x}$ is simply

$$- c_{n,n-1} + c_{n,n} = 0 = \lambda \tag{A9}$$

implying that zero is an eigenvalue of C and hence a contradiction because C is a positive definite matrix.