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A Survey of CAD Model Simplification Techniques for Physics-based Simulation Applications

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Abstract

Automated CAD model simplification plays an important role in effectively utilizing physics-based simulation during the product realization process. Currently a rich body of literature exists that describe many successful techniques for fully-automatic or semi-automatic simplification of CAD models for a wide variety of applications. The purpose of this paper is to compile a list of the techniques that are relevant for physics-based simulations problems and to characterize them based on their attributes. We have classified them into the following four categories: techniques based on surface entity based operators, volume entity based operators, explicit feature based operators, and dimension reduction operators. This paper also presents the necessary background information in the CAD model representation to assist the new readers. We conclude the paper by outlining open research directions in this field.

1. Introduction

Physics-based simulations play an important role during the product realization process. Let us consider few representative examples. Multi-body dynamics simulations are used to determine the sizes of actuators during the design of robots. Finite element simulations are used in structural and thermal analysis of components in the automotive and aerospace industries. Computational fluid dynamics simulation is used in automotive engine cooling system design. These simulations help in reducing the need for expensive physical prototyping and hence shorten the product development time and reduce the product development cost. Apart from these design examples, physics based simulation is also used in assembly planning, ergonomics analysis, and testing applications.

Physics-based simulations are primarily driven by 3D CAD data. The computational performance of simulations depends on the number and complexity of the geometric features present in the CAD model. Features are an integral part of modern CAD model and they are used in virtually all the domains of product life cycle, namely design, manufacturing, analysis and maintenance. Even the presence of a single, relatively small geometric feature can increase the size of the underlying discrete physical simulation problem by as much as 10-fold [Whit03, Lee05a]. If we run a finite element analysis on a part with hundreds of small features as shown in Fig.1, the computational time will be very large. Extremely large computational times limit the usefulness of simulations during the design cycles. Complex models may often lead to ill-conditioned matrices and hence working with non-simplified complex models may produce inaccurate results [Saad03]. Hence, simply utilizing more powerful computers will not solve the

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problem associated with highly complex models. In order to get accurate results in a timely manner, one must utilize simplified models that retain the important details and eliminate the irrelevant ones.

To illustrate the above mentioned point, let us consider a simple example of a part (see Fig. 2) subjected to different kinds of simplifications. Fig. 2(a) shows the solid model of an axis-symmetric part with several grooves and holes. Fig. 2(b) shows the simplified part model with the tiny grooves and holes removed, which can be used for an application like rigid body simulation where small holes and grooves play a negligible role in determining the inertia tensor and the collision contact points. Fig. 2(c) shows a simplified 2D model exploiting the symmetry of the part, which can be used for an application such as thermal analysis. Fig. 2(d) shows the simplified part composed of a beam and a plate element which can be used in structural analysis. All these simplification instances reduce the computational time significantly while affecting the respective simulation results negligibly as compared to the full blown solid model. Currently, such kinds of feature simplifications and idealizations are mostly performed manually. Manual feature simplification, however, requires human expertise and is time-consuming.

Several efforts have been made over the last few years to automate the model simplification process. It is studied in various contexts such as finite element analysis and collision detection within the overall category of physics based simulation [Arms94]. In the collision detection field, some of the reported work relates to dynamic simplification [Yoon04] and construction of bounding volume hierarchies [Tan99]. These primarily relate to multi-resolution representation for performing collision detection at various levels and do not address the problem of model simplification explicitly and, thus, will not be covered in this paper for the sake of brevity. Polygonal mesh simplification has been extensively studied by the graphics community [Cign98, Lueb01, Lueb02]. We will not focus on the model simplification methods for graphic rendering in this paper e.g. [Elsa98]. We have, however, covered some techniques that address model simplification in the context of network model transmission from rendering and occlusion analysis perspective and can also be applied to collision detection [Andú02, He95].

Techniques developed for different contexts have different simplification objectives and hence different simplification outcomes. The purpose of this paper is to compile a list of techniques that are relevant for physics based simulation problems and to characterize them based on their attributes. The remainder of the paper is organized in the following manner. In Section 2, the basic terminology used in model simplification area is explained in details. Sections 3 through 6 describe the model simplification techniques. It is difficult to discuss all the reported papers in details and thus for the sake of brevity, we have described only few representative techniques in each category in detail. We have included references to the remaining techniques in each category. Section 7 discusses a systematic taxonomy of the covered techniques and presents several criteria to aid the readers for selecting a model simplification technique suiting their requirements. Finally, Section 8 concludes this paper by highlighting areas of open research.

2. Terminology

Many different schemes are used to represent 3D geometric information. The first scheme stores the boundary information for a solid (i.e. vertices, edges and faces together with the connectivity information) and is popularly known as the boundary representation (B-Rep). Another scheme stores the history of applying Boolean operations on a solid and is called a constructive solid geometry (CSG) representation [Lee99, Shah95, Hoff89]. The third scheme stores solids as an aggregate of simple solids such as cubes (voxels). Solid models described in this manner are known as decomposition models. The fourth scheme explicitly stores feature information in addition to the information about the elementary shape entities (vertices, edges, faces etc.) and is referred to as feature based modeling. Features can be classified into two main types - primary, volumetric features such as holes, pockets, slots etc., and secondary, surface features like blends, fillets and rounds. Secondary features are usually introduced in industrial parts to smoothen the sharp edges of the part to enhance strength, aesthetic appeal, handling safety and ease of manufacturability. The model simplification operators operate on the model representation and perform simplification. Based on the type of simplification operators, the techniques for model simplification are categorized into surface entity based, volumetric entity based, explicit feature based and dimension reduction based type in the current paper.

In case of surface entity based techniques, features are simplified by operating on surface entities such as faces, edges and vertices. Under this category, reported techniques are edge collapse technique [Lee02], face clustering [Shef97, Shef01] and low pass filtering approach such as Fourier Transforms [Lee98]. In case of volumetric entity based techniques, the features are simplified by operating on volumetric entities from the model. The major techniques reported under this category are effective volume [Lee05a, Choi02], cellular representation [Lee04, Lim04], and voxel [He95, Andu02] based approaches. In the explicit feature representation based techniques, the feature information and semantics is explicitly extracted from CAD model or is present in the model in addition to geometric and topological data. The features are user defined and depend on the context of application (*e.g.* in case of mechanical analysis holes, slots, steps, pockets, fillets, rounds, etc. are the sought features). The feature information is used and operated upon unlike implicit model representation schemes where only geometric and topological data is directly used for model simplification. Feature based modeling and feature recognition techniques are described in details in [Shah95]. The reported methods are explained in Section 6. The dimension reduction scheme deals with expressing models by idealizing the features into reduced dimensional shapes [Lee03, Don96, Don00, Reza96]. The reduced dimensional shapes are application dependent. For finite element analysis, the model features are idealized into one-dimensional beams, point masses, etc. Thus, the dimensional reduction operators convert a model into a reduced dimension model or mixed dimension model based on the level of abstraction desired [Robi06]. In a mixed dimension model, the model is constituted of higher as well as lower dimension features. The remainder of this section introduces underlying foundations for describing CAD model simplifications approaches.

Non-manifold Topology (NMT): NMT is a model representation scheme introduced to address the problems such as absence of multiple representations for concurrent engineering and expensive Boolean operations faced by conventional model representation schemes. It is a representation of wireframe, surface and solid models in a single architecture [Masu93]. Conventional CAD data structures (*e.g.* B-Rep) can represent only 2-manifold objects. For 2-manifold objects, neighborhood of any point on its boundary is homeomorphic to an open disc in $\Re^2$ (two dimensional Euclidean space). An object for which the above condition is not satisfied is called non-2-manifold topology or simply NMT [Chen93]. NMT is useful to represent mixed dimensional models (obtained after dimension reduction as explained in section 6), stand-alone faces, wireframe edges etc. which cannot be represented using conventional CAD data-structures. Several data structures have been reported that implement NMT, out of which cellular complex and partial entity structure are important from model simplification point of view. Fig. 3 shows some non-manifold cell complexes.

Cellular Representation: Cellular representations have been used for identifying model simplification opportunities [Bida93, Bida98, Bida05, Gome93]. Cellular representations capture both positive and negative spatial regions, and may be viewed as curved voxelized spaces where the exact boundary representation is embedded in the cell boundaries. They are non-manifold geometric representations of the feature model of a product and consist of cell complexes [Cava97]. A cell complex can represent geometric shape relevant to engineering domain effectively, as it includes wireframe, surface and solid models or a combination of them. Mathematically, a cell complex (C) is defined as a set of *n*-cells, where an *n*-cell is a bounded subset of 3-D Euclidean space that is homeomorphic to an *n*-dimensional open sphere. It satisfies the following properties [Masu93]:

i. 3D cell complex can be represented by a collection of 0-cell (vertex), 1-cell (edge), 2-cell (face) and 3-cell (volume).

ii. The whole boundary of each element consists of lower dimensional elements.

iii. No two topological elements intersect each other.

Thus, a cell complex represents parts as a connected set of volumetric disjoint cells of arbitrary shape, and represents each feature as a connected subset of these cells [Bida98]. The cells defined above represent the part shape exactly. In existing approaches for converting features to cellular representations, the subdivision is determined by the property that two cells may never overlap volumetrically. So, whenever two features overlap, their cells are subdivided such that one or more cells are shared by the two features and the remaining cells belong to either one of them. In a cellular representation, a part is represented as a cellular decomposition of the space, where each cell in the space is either void or filled. Filled cells represent the material that belongs to the part. The void cells represent the surrounding space. An example of a cellular decomposition is shown in Fig. 4.

The transformation operations on cells are represented using the concept of cell maps [Lee05a, Ragh98]. In one of the earlier approaches, Bidarra *et al.* discussed application of the concept of cellular complex for developing feature addition and removal operators from the cellular representation of a model [Bida98]. A feature model is partitioned into nonintersecting cells. Each cell stores the list of features to which it belongs to, referred to as the cell's *owner list*. The two modification operators defined on the cellular model in this work are insertion and removal of feature shape. The two main effects of any of these operators on the cellular model are topological modification and owner list modification.

In case of insertion of a feature, either the cells just touch each other (no new cell is created in this case) or overlap volumetrically (a new cell is created). The new cell thus created inherits the owner list from its originating cells. In case of feature removal, the cells and their references are removed from the owner lists of other cells. The resulting cells are checked for their owner lists. If the owner list for a cell is found to be empty, that cell is also removed. The cellular representation scheme and the operators developed are useful in handling feature interactions in various applications such as multi-view modeling and feature visualization. This is a general approach for feature addition and removal in a cellular model that can be used for model simplification in addition to multi-view modeling related applications.

Effective Volume Operators: Feature rearrangement based on given criteria for Level of Detail (LOD) and Level of Abstraction (LOA) is required for detail removal, leading to a resulting shape different from the original feature shape owing to the non-commutative nature of the union and subtraction Boolean operations. The operators involving combination of Boolean operations giving geometrically and topologically valid resulting features after rearrangement are called effective volume operators. Let V_i denote the volume of the solid primitive of a feature F_i, $\otimes_i$ denote a Boolean operation, and M_n denote the resulting model obtained by applying n Boolean operations between n+1 solid models: [Lee05b, Lee05c, Lee05d, Lee06a, and Lee06b].

$$M_n = \prod_{i=0}^{n} \otimes_i V_i, \quad where \otimes_0 V_0 = \phi \otimes_0 V_0 \tag{1}$$

where, ϕ is an empty set.

If the j^{th} Boolean operation $\otimes_j V_j$ is moved to m^{th} position, M_n can be represented as follows:

$$M_n = \left(\prod_{i=0, i \neq j}^{m} \otimes_i V_i \right) \left(\otimes_j \left(V_j - \sum_{l=1}^{m-j} \varphi\left(\otimes_j, \otimes_{j+l}\right) V_{j+l} \right) \right) \left(\prod_{i=m+1}^{n} \otimes_i V_i \right) \tag{2}$$

where,

$$\varphi(a,b) = \begin{cases} 0, & if \ \ a = b \\ 1, & if \ \ a \neq b \end{cases} \tag{3}$$

Equations (2) and (3) can be explained by the following example [Lee05b]. Fig. 5 shows a sample NMT model created by applying five form features. If the order of features is changed to $F_0 \to F_2 \to F_3 \to F_4 \to F_1$ by moving F_1 to the last location, the Boolean operation sequence corresponding to the new feature order will be $P_0 \cup P_2 - P_3 \cup P_4 - P_1$. However, if Equations (1) to (3) are applied, the effective zones of F_0, F_1, F_2, F_3, F_4 are P_0, $P_1 - P_2 - P_4$, P_2, P_3, and P_4 respectively. The Boolean sequence becomes $P_0 \cup P_2 - P_3 \cup P_4 - (P_1 - P_2 - P_4)$, and its result is the same as the original part shape as shown in Fig. 6. Note that the dashed line in the figures represents a hole generated by subtracting a wireframe from a solid.

Voxels: A voxel is a volume element, representing a value on a regular grid in three dimensional space. Voxels can be understood as the three dimensional equivalent of a pixel, which represents two dimensional image data. They are frequently used for visualization and analysis of volumetric data commonly occurring in CAD/CAM, bio-medical, scientific data visualization, etc.

Partial Entity Structure (PES): PES is a compact and fast model representation scheme. It is developed as a non-manifold B-Rep data structure. The topological entities in PES are of two type namely, primary and secondary entities. The primary topological entities contain 0-cells (vertices), 1-cells (edges), 2-cells (faces) and 3-cells (regions) and their bounding elements (i.e., loops and shells). The secondary topological entities consist of partial vertices, partial edges and partial faces. The partial vertex represents the non-manifold case where more than one two-manifold surfaces are connected to the vertex. A partial edge represents the non-manifold condition where more than two faces are connected to an edge. A partial face is used to represent the non-manifold condition where a face is adjacent to two regions [Lee01]. The secondary entities are called *partial topological entities* or *partial entities.* Geometrically, partial entities represent adjacency relationships among the primary entities. Fig. 7 illustrates the different partial entities.

Medial Axis Transform (MAT): The MAT was initially proposed by Blum as a technique for biological shape measurement [Blum67]. It provides a skeleton-like representation of the shape of a model, based on the geometric proximity of its boundary elements. MAT can be described as the locus of the center of a maximal inscribed disk as it rolls around the object interior, where a disk is maximal if it is contained within the object but not within any other disk [Dona00]. Concept of Medial Axis can be easily translated into 3-D domain to Medial Surface by replacing circle with sphere in the former definition. Concept of MAT is very useful in generating FEA idealization models as it provides geometric and topological proximity information enabling appropriate idealizations to be identified. A 2-D model can be dimensionally reduced to a 1-D beam and a 3D solid can be replaced with a 2D surface, which is comparatively computationally less expensive to analyze. Fig. 8 shows a channel section and its medial axis transform.

One of the modifications to MAT is θ-MAT which is a subset of MAT. For a θ-MAT, separation angle for each point on the MAT is greater than the specified angle θ. Separation angle for a point on an MAT is defined as the angle subtended on it by its nearest neighbors on the polyhederal model. Thus, a θ-MAT for a model approaches the MAT for the model as θ approaches zero. The θ-MAT of a model is computationally more stable than its MAT as it is not affected by fineness of tessellation of the model.

Surface Simplification by Decimation: Decimation is a class of technique to simplify the surface geometry of a mesh model by removing or simplifying topological entities (vertex, edge or a face) and making changes in the model to retain the topology of the overall model. In case of vertex decimation, a vertex along with its associated triangles is removed and the hole created is retriangulated to maintain the topology. In case of edge decimation (or edge collapse), edges are collapsed into vertices. In case of face decimation (or face collapse), faces are collapsed into vertices. The entities are selected for removal based on predefined error metrics that evaluate the geometric impact of their removal. One of the reported error metrics used is quadric error [Garl97, Lee04]. Using the fact, that each vertex is an intersection of several planes, the quadric matrix Q (a 4 x 4 symmetric matrix) is defined for very vertex by aggregating the coefficients of equation of the planes in a way such that the quadric error (e) associated with the given vertex $\bar{v} = [v_x, v_y, v_z, 1]$ is given by a quadratic form as shown in Equation (4).

$$e = \bar{v}^T Q \bar{v} \tag{4}$$

The quadric error (e) can be used for ranking the vertices for consideration of decimation. In the particular instance of edge collapse ($\bar{v}_1, \bar{v}_2 \rightarrow \bar{v}$), the quadric corresponding to $\bar{v}$ is calculated as sum of quadrics associated with $\bar{v}_1$ and $\bar{v}_2$. The approach is quite general and can be applied to face decimation as well, by determining quadric matrix for face using summation of quadrics associated with vertices defining the face. The decimation approaches were originally introduced for obtaining multi-resolution models by simplifying the entire model at a time. Recent techniques involve selective simplification of surface artifacts using the same approach but limiting the simplification process in the region of interest [Lee04].

3. Techniques Based on Surface Entities

Boundary representations describe objects in terms of surface entities. Hence, many researchers have developed techniques to perform simplification using surface entities. The techniques where surface artifacts of the given geometric model are simplified are referred to as techniques based on surface entities in this paper. These reported techniques fall into the following three subcategories – low pass filtering, face cluster based simplification and size based entity decimation. We discuss reported work in each category in the following subsections.

3.1 Low Pass Filtering

The low pass filtering using Discrete Fourier Transform (DFT) is a technique based on operators acting on surface entities [Lee98]. This model simplification technique is applied mainly to Finite Element Analysis (FEA) mesh generation. A shape needs to be digitized before performing discrete Fourier Transform on it. The input model is in the form of a 2-D grayscale digital image of resolution 512 x 512. The black pixels represent the locus of points satisfying Equation (5) and the white pixels represent the surrounding space.

$$\left.\begin{aligned} h(x,y,z) \geq T_0 \\ h(x,y,z) < T_0 \end{aligned}\right\} \quad \begin{aligned} &if\ (x,y,z) \in V \\ &if\ (x,y,z) \notin V \end{aligned} \qquad (5)$$

where h represents the surface of the model, V represents the volume occupied by the model and T_0 is the threshold representing model boundary. The black pixels are assigned a value of 1.0, whereas the white pixels are assigned a value of -1.0. The transition pixels where the value switches from -1.0 to 1.0 are assigned a value of 0.0 to represent the model boundary. Using these values as the height, a surface can be plotted. After this, Fourier transform is applied to the surface function of the part model. The surface is thus, expressed in the frequency domain, after application of Fourier transform.

Authors have stated that the low frequency terms constitute the overall shape while the high frequency terms represent the detailed features in the transform of the surface function of the part model. When the high frequency components are removed from a model, it is called as low pass filtered (LPF) model. In the LPF model thus generated, the sharp edges are converted into smoothed edges. The smoothed edges are not desirable in analysis model as they suppress the important effect of stress concentration. To get the feature suppressed models with sharp edges retained, the original *unsimplified* model is compared with the LPF model. The average distance between the edges and faces of the original model and LPF model is evaluated as a metric to denote the complexity or *detailness* of the respective entity. If the metric is below a specified value, the entity is considered detailed and if the metric is greater than the specified value the entity is considered to belong to the overall shape. Authors have also explored the possibility of using wavelets instead of Fourier basis function in this method of low pass filtering. Wavelets have narrow support and, thus, are not suitable for removing detailed features. All the small features influence the overall shape of the model. Thus, the basis function describing a small local feature influences the entire domain rather than a region lying in its vicinity. Use of the metric ranks the features according to size and acts as a measure for determining the order of features to be suppressed, as it represents the deviation between the original model and the filtered model. The computational time for mesh generation is significantly reduced by using this method. It is also considered as a potential technique for feature recognition and is reported to be robust in suppressing details even for complex geometries. The method is not fully automated and some human intervention is required to select the features to be removed. The method is currently developed only for 2D surfaces and not extended to 3D volumes. The methodology is also limited in terms of filling the gaps (the removed faces) if the method is extended to 3D models.

3.2 Face Cluster Based Simplification

The face clustering algorithm reported by Sheffer is a technique to cluster the faces in the input model [Shef01]. The clusters thus formed represent regions of interest that may be considered for simplification. The method is mainly developed for simplifying the models for FEA mesh generation application. There are three main steps followed in this approach: face clustering, finding the collapsible faces and simplification. The model is represented as an adjacency graph with all the faces represented as initial cluster nodes and connecting edges as arcs. The arcs are then contracted to cluster the faces. The face node pairs to be clustered are selected based on the weights assigned to the arcs joining them. The weight depends upon the geometry indices denoting the compatibility criteria (determined heuristically) of non manifold input.

Once the face clusters are made, a collapsibility check is done for all the face clusters. The collapsibility check depends upon criteria like boundary preservation, region size, region smoothness and simplicity of region boundary shape. The metrics for each criterion are defined mathematically in this paper. These metrics are the error measures for evaluating the effects of removing the clusters on the overall model geometry. Finally, decision is taken by the algorithm about which clusters need to be collapsed based upon the metrics evaluated for each cluster. Collapse operators are defined based on virtual topology, where a face is split into different faces equal to the number of adjacent neighboring faces sharing a boundary with the original face (the neighboring faces) [Shef97]. After this, the newly created faces are merged with the respective neighboring faces with which they share the boundary. It can be seen that only the connectivity between the faces changes; however, the geometry is preserved and hence it is called a virtual topology based collapse.

The advantages of the above described approach are its applicability to faceted, free form geometries as well as non manifold models. A curvature-based index is introduced in this paper which can be used for handling curved regions. The curvature-based index is used in the cases where the geometry of clusters is non-planar and planer index cannot be used. The method is applied on several example parts and it is found that the simplified models have significantly fewer elements without loss of accuracy. This is because the simplification process only changes the connectivity but the original geometry is maintained. However, the ranking method for clustering the faces is not systematic and is based on heuristic criteria. Also, since the face clustering algorithm is of the greedy type, the resulting clusters are not necessarily optimal.

Inouea *et al.* reported a face clustering technique for FEM mesh generation application similar to virtual topology [Inou01]. The approach iteratively merged the model faces to obtain face cluster regions having sufficiently large area compared to the mesh element size, smooth enough face boundary and flat surface. Authors defined metrics for mesh area, boundary smoothness and surface flatness and used them for ranking the faces for merging purpose. Dey *et al.* reported local modification of automatically generated meshes [Dey97, Shep98]. They defined *a priori* error metrics based on aspect ratio and dihederal angle measure and suggested model simplification by iteratively removing elements with poor aspect ratio and small angle metric and remeshing. In their approach, the validity of locally modified mesh is ensured by imposing topology based constraints.

3.3 Size Based Entity Decimation

Lee *et al.* present a method to generate progressive solid models (PSM) from feature based models using a cellular topology-based approach [Lee04]. Here cellular topology is used for generating the PSM and then surface entity based operators are developed to simplify the model. The main concept in this paper is to start with a feature based model as input and generate a sequence of solid models representing the underlying object with various level of details. The intended purpose of PSM is to stream models over a network efficiently.

A PSM is defined as an overall shape and a set of details. Authors argue that the problems with traditional approaches in generating such progressive model is the high computational cost associated with applying Boolean operations for transitioning between levels of detail and the cost of storing the various shapes. The authors have come up with cellular topology representation of part model to overcome these difficulties in PSM. Feature shapes have an explicit volumetric representation in terms of cells. PSMs can thus be derived from the face based composition and decomposition of cells that eliminate expensive Boolean computations and storage space. In contrast to the spatial representation schemes (voxel based), cell-based representations are exact in nature. The delta volumes are represented using progressive features in case of cell based technique. The features are simplified or suppressed to produce PSMs. The simplification of features is done using edge contraction techniques. The edges in a feature are ranked based on the geometric error introduced by removing them. The edges are then contracted one by one starting from the lowest cost edge. The positive features are suppressed only using the criteria of feature volume. If the volume of feature is below a certain threshold value, the feature cells are attributed to be “dummy cell” initially. Later, they are considered as positive cells when the model resolution is increased.

The approach described above can handle the exact NURBS representation of the underlying models. The Boolean operations for progressive modeling are replaced by simpler topological entity manipulation at cell level, which is computationally much better. The cell and feature simplification criterion is mainly designed for solid model transmission. Complexity of features or the importance of features with respect to a particular application is not considered in this technique as model simplification criteria.

The cell based transformations are used for transforming higher dimensional cells to same or lower dimensional cells e.g. from face to edge. The techniques where operators defined using such cellular transformation acts on surface entities like faces, edges and points for simplification, are studied under the surface entity based category in this paper. This technique uses cell to cell transformation functions for simplifying models by suppressing features for Finite Element Analysis model preparation [Lee05a]. Authors have presented a general methodology to suppress or reinstate features from a B-Rep model by using invertible cell to cell mapping. The cell to cell mapping functions are implemented in the CADFix software in terms of three surface based operator pairs namely collapse/explode, split/join and insert/remove [CADf05]. In case of collapse operation, the transformation of a cell to a lower dimension takes place. For example, a two-dimensional face is collapsed into a one-dimensional edge. The explode operation is defined as inverse of collapse operator based on collapsing history i.e. the information about collapsed face is used to reinstate the same face when the collapsed edge is exploded.

In case of a split operation, a cell within a model having dimension greater than zero (*i.e.* a point), can be split into two or more cells of similar dimension by introducing one or more cells of lower dimension. For example, a face (2D cell) can be split into two faces by introducing an edge (1D cell). The splitting cell is constrained to lie within the parent cell, so that an edge partitioning a face lies on the face. The same logic applies to splitting an edge by introducing a vertex and a region by introducing a face. Joining two faces again uses splitting history and combines the faces split earlier. In case of an insert operation, lower-dimensional cells are added in the interior of a cell. For example, a vertex can be inserted on an edge. The insert operator is useful in CAE when cellular partitioning is required to represent loading and boundary conditions. In the remove operation, a lower dimensional cell is deleted from the higher dimensional cell. For example, a loop representing hole on a face can be removed using this operator.

To suppress features from a B-Rep model, it is first expressed in cellular representation and then the cells are mapped into simplified model by using the operators described above. One of the important requirements in analysis models is to suppress the narrow regions. A narrow region is defined as part of surface where two of the boundary edges come in close contact [Lee03]. The narrow regions pose problems in mesh generation because of huge size differences within the region leading to poor element quality. To suppress the narrow regions, the edges in proximity are identified and then the face is split at each end of the region. The resulting long narrow face is then suppressed by collapsing the short edges to vertices and merging the two long bounding edges. No error measure or threshold has been defined to select the features to be removed. Instead, they are selected interactively by the user for simplification purposes.

The main advantage of this methodology is the generality of its implementation. The method can be implemented by explicit cellular representation or direct geometry based operators. An "audit trail" or analysis history is generated that is useful for analysis to compare various simplification strategies thus obtained by the system. Narrow regions can be suppressed using this approach, which is important for mesh generation. A possible limitation is that the faces are repartitioned in this approach which requires some post processing for generation of meshes of desired density.

Focault *et al.* reported a topology simplification technique for finite element mesh generation [Foca08]. The model for FEA should be prepared in such a way that the mesh generated for it represents the model as closely as possible and at the same time minimizes the computation time for analysis. Another requirement of the simplified model is that the mesh should represent the boundary condition domains, e.g., a point on the part where forces or displacements are applied should be represented by a coinciding node. Also, the mesh edges generated from the simplified model must exactly match the sharp corners of the geometry to minimize the discretization error.

The authors have developed a Mesh Constraints Topology (MCT) based model simplification scheme to address the sharp corner matching requirements. MCT entities are defined as composite topological entities created to suit mesh generation requirements stated before. The MC face is a poly-surface, defined as the union of Riemannian surfaces constituting the reference model. The MC edge is a poly-curve, defined as union of Riemannian curves constituting the reference model. The MC entities are used to represent the model because they preserve the exact geometries as they are a higher level representation of the low level B-Rep entities that define the reference model. The MCT models are internally represented using hypergraphs. Hypergraphs are graphs with arcs that connect two or more nodes. MCT is defined using three topological adjacency hypergraphs namely, face-edge adjacency, face-vertex and edge-vertex.

The authors have implemented graph based operators for deleting MC edge and vertex, inserting MC vertex in MC edge, collapsing MC edge to MC vertex and merging MC vertices for simplification. The mesh quality constraint for MC entities is decided by their size and curvature. The size for an MC face is defined as distance between face boundaries while for an MC edge as length of the edge. The size of MC entities must be greater than a set size threshold while curvature (leading to deviation angle) less than a set curvature threshold to satisfy the mesh quality constraint. The deviation angle is the angle between the normals of adjacent mesh segments for a given discretization error. For enlargement of thin MC faces, MC edge deletion operator is used. Redundant edges situated in planar regions need to be deleted using MC delete operator. MC vertex deletion and MC edge collapse operators are used to get rid of small MC edges. Constricted sections in MC faces are suppressed by using MC vertex insertion and vertex merging operators. Thus, the MCT operators and criteria along with the mesh quality constraints are used together to simplify the models. The mesh quality constraints used by the authors are mesh element size and discretization error. The mesh element size is represented using over density ratio which is the ratio of the size map to the effective element size. The discretization error is a measure of the gap between the mesh and the geometry. The simplification method described here, retains the topology of the model and thus no defeaturing errors are introduced after simplification. However, FE numerical errors because of changes in the geometry are introduced and that is controlled by refinement schemes.

The method described above retains the topology and creates new geometry adapted to mesh quality constraints such as size-map, maximum over-density, maximum deviation angle, and boundary condition zones.

Fine *et al.* introduced idealization operators for Finite Element Analysis [Fine00]. The operators are based on vertex removal and spherical error zone concept. Mobley reported an object oriented approach to develop surface based defeaturing operators to suppress small features for FEA model preparation [Mobl98]. Date *et al.* reported vertex and edge collapse based technique for mesh model simplification and refinement [Date05]. They defined three metrics based on overall geometry error, face size and face shape. The metrics are evaluated for edges to determine their priority index for simplification. Veron and Leon reported shape preserving simplification for a polyhederal model using vertex decimation [Vero98]. A metric based on discrete Gaussian curvature and angle subtended by incident edges is evaluated for each vertex in the model. The vertices are then removed based on their rank priority based on above metric. For example, the vertices with curvatures near zero denote that they lie on plane and are removed first. The vertices with curvatures farther from zero (both in positive and negative directions) are considered later for removal.

All of the techniques listed in this section are summarized in Table 1.

4. Techniques Based on Volumetric Entities

The techniques where volume based artifacts are removed for model simplification are called volume entity based techniques. Such techniques are further classified into two subcategories – voxel based and effective volume based techniques.

4.1 Voxel Based Simplification

Andújar *et al.* proposed Trihederal Discretized Polyhedra Simplification (TDPS) based topology reducing surface simplification technique [Andu02]. The aim of this work is finding a reduced valid polyhedral approximation of a solid model such that the maximum solid Hausdorff distance is below an error value. There are three steps followed in this approach: discretization, reconstruction and face reduction.

The model is discretized using Maximal Division Classical Octree (MDCO). MDCO subdivides the cubic universe into eight octants and arranges it into an 8-*ary* tree, the leaves of which are labeled black or white based on whether they fall completely in or out of the model volume respectively. Nodes that fall partially in the model volume and partially out of it are labeled as gray. The black and white nodes are not subdivided further, whereas the gray nodes are subdivided till their parts become white or black or the depth of the tree reaches a set level. In the reconstruction step, the MDCO is compared with the original model and polyhedral surface (*S*) based on the solid Hausdorff distance as error measure is extracted. The surface (*S*) satisfies the conditions that all the black nodes are completely within *S* and all the white nodes are completely outside *R* and all the border nodes (defined as nodes whose at least one of the twenty six neighbors is white) intersect with *S*. Finally, in the face reduction step, the surface obtained from the previous step representing the original model is further simplified using edge collapse technique.

Although it appears that the simplification operator used is surface entity based, the error measures and topology preserving simplification strategy are based on volumetric operators. Hence we have chosen to list this method under volumetric entity based model simplification category. The major advantage of this method is its applicability on individual parts as well as assemblies. Also non manifold inputs can be effectively handled by this approach to produce 2-manifold solids.

One of the limitations in the methodology described above is the performance issue. Small error thresholds require larger subdivision levels for the MDCO. The cost of TDPS largely depends on the subdivision level of MDCO and thus volume based technique is computationally intensive for small error thresholds. Another limitation is the requirement for preprocessing of input model before simplification to align it with the axes so that octree represents the part more accurately. This is because of the isothetic nature of the octree. TDPS method is applicable to collision detection, occlusion analysis, multi-resolution robust Boolean operations, indirect illumination, acoustic modeling and query acceleration.

One of the earliest approaches based on operators acting on volumetric entities is reported by He *et al*. [He95]. They developed a voxel based object simplification technique using marching cubes algorithm to generate multi-resolution triangle mesh hierarchy.

4.2 Effective Volume Based Simplification

A feature based non-manifold modeling system is developed by Lee *et al.* to address the needs of both CAD and CAE applications simultaneously from a single model [Lee05b, Lee05c, Lee06a, Lee06b]. This system supports feature based multi-resolution and multi-abstraction modeling capabilities. The main advantage of non-manifold model is its capability to represent any combination of wireframe, surface, solid and cellular models in a unified data structure. Partial entity structure is used to store the model information [Lee01]. The detail removal and dimensional reduction at various levels of detail and abstraction requires features to be rearranged. This rearrangement of features may result in different final shapes owing to the non-commutative nature of the union and difference operators. Authors have developed the concept of effective volume (explained in Section 2) of features and presented theorems for exchange and rearrangement of features.

The detail removal process involves three steps that are briefly stated as follows. Firstly, all the idealization features having application domain as design are extracted from the master model. Secondly, the extracted idealization features are rearranged according to LOD criteria. The LOD criterion selected by authors is based on decreasing volumes. If the volume is below a threshold, the corresponding feature is considered for simplification. Thus, the error measure for selection of features for simplification is the volume of the feature. Finally, the LOD is set interactively to remove the features below the specified level. The multi abstraction of the model on the other hand is performed by applying LOA criteria to abstract the features. LOA criteria are application dependent. In case of structural analysis applications, aspect ratio of the feature is used to set the abstraction level. In case of injection molding simulation application, the mesh size is used to set the abstraction level.

The approach described above is particularly advantageous for multi-resolution modeling and capable of LOD simplification of features. The model representation scheme using partial entity structure enables use of the same model in extracting CAD and CAE information leading to CAD-CAE integration. The model editing operations consisting of a given sequence of Boolean operations is carried out for a CSG tree containing only a subset of the primitives in the merged set, and selectively filters out these primitives from the final result without actually removing them from the merged set. The feature based modeling capabilities for both feature deletion and feature interaction detection are thus computationally efficient and simple.

Some of the limitations of the approach described above are explained as follows. When the shape or semantics of a feature are altered due to feature interactions, the predefined abstract model of the feature may become invalid. Authors have pointed out that this problem can be solved by adopting automated medial axis/surface transformation method. The physical constraints and properties such as boundary conditions and mass properties applied to CAD models have also not been transferred to CAE models.

All of the techniques discussed in this section are summarized in Table 2.

5. Techniques Based on Explicit Features

Many reported model simplification techniques are based on recognizing explicit application features before simplification. These techniques define class of explicit features related to a particular application such as manufacturing, FEM etc. and evaluating some metric based on which simplification decision can be taken. The techniques fall into the following subcategories – prismatic feature simplification, blend simplification, and arbitrary shaped feature simplification based on the type of features covered.

5.1 Prismatic Feature Simplification

The three steps followed in the model simplification from polygonal mesh by Date and Nishigaki includes feature recognition from the input mesh, mesh simplification and feature recovery [Date06]. The steps are explained as follows.

In the feature recognition step, mesh segmentation technique is used. The features considered in this work are blind holes, through holes and bosses. The dihedral angle between two faces shared by the common edge is used for extracting the feature edges. These sets of edges are used to extract regions of interest and segment the mesh into regions. The regions with area larger than the threshold set by a user are classified as base surfaces. After this, the triangles that are not coincident with base surfaces are extracted as Feature Construction Triangles (FCT). The feature type identification and feature parameter extraction is then done using three rules: the FCTs with two loops are identified as through holes, the FCTs with one convex loop are identified as blind holes and the FCTs with one concave loop are identified as bosses. The feature parameters are extracted by fitting a least square plane on the feature boundary and finding the depth, width and height for each kind of feature. The location of each feature is determined using the centroid of feature boundary on the least squares plane.

In the mesh simplification step, edge collapse is used to eliminate the vertices that are redundant. The main consideration during edge collapse is to keep correspondence between the feature boundary edges (FBE) and the boundary of feature meshes so that features are recoverable. This is done by creating a metric to evaluate the approximation error of FBEs and using it to decide whether the edge needs to be collapsed.

In the last step of feature recovery, the feature removing triangles (FRT) are replaced with feature FCTs to reinstate the suppressed features. The condition that has to be satisfied in order to recover a feature is that the feature boundary vertices (FBV) should match with the boundary vertices of the removed feature mesh. If this condition is not met, then the local LOD method is used. In the local LOD method, the mesh is represented as a binary tree containing a set of parent nodes representing the collapsed nodes and corresponding children nodes consisting of the vertices that are collapsed to make the parent node. This binary tree is traversed to recover neighborhoods iteratively by vertex split. The features suppressed can be recovered based on the Level of Detail tree.

The approach described above is capable of simplifying simple features (with one base surface) like blind and though holes and bosses. However, nested features such as hole in a pocket cannot be simplified.

Another reported technique to recognize and suppress features for finite element model preparation is given by Ribelles *et al.* that utilizes face clustering technique for feature recognition [Ribe01]. Feature based methodology for finite element model idealization has been discussed by Dabke *et al.* by extracting axis-symmetric, plane and solid features from input B-Rep model [Dabk94].

5.2 Blend Simplification

The approach by Zhu *et al.* deals with simplification of fillets and round features in a B-Rep model [Zhu02]. The main application of model simplification in this work is to prepare the model for further feature recognition purposes by removing fillets and rounds. From the input B-Rep model, convexity of all the topological entities namely faces, edges and vertices are characterized, which in turn is used to identify the trace faces. Trace faces are the smooth faces obtained by blending sharp edges while modeling. The trace faces are then used to identify the fillets and rounds in the model. After identifying them, the topological entities pertaining to these features are cleaned and the gaps thus produced are filled up by a method developed by the authors named as incremental knitting process.

The authors have underlined three main considerations while suppressing the fillets and round features, namely topological consistency, geometric consistency and reversibility. By preserving the topological consistency, it is meant that the resulting model should be a manifold model. When the fillet faces are removed from a model, gaps are created. These gaps may make the model topologically invalid. The gaps should be filled by topological entities to keep the model topologically valid. By geometric consistency, it is meant that the edge replacing the fillet surface should lie on the planes of supporting faces of the suppressed fillet. Reversibility means that the attributes of the suppressed fillet should be preserved to roll back the model if needed.

The trace faces are classified by the authors into three types as toroidal, cylindrical and spherical. The toroidal faces are generated by blending circular edges. The cylindrical faces are generated by blending straight edges. The spherical faces are generated by blending a convex vertex. Several trace faces are connected together to form face chains. Face chains forming closed loops are called closed face chains; otherwise, they are known as open face chains. Homeomorphic equivalence is used to map the face chains to a disc or a ring. The disc or ring representation is used to test the occurrence of gaps when fillet faces are removed. The gaps are then knitted using KR (knitting on ring) or KD (knitting on discs) algorithms developed by the authors. KR and KD algorithms are used to introduce new topological entities after removing fillets to maintain topological and geometrical consistency. To ensure the reversibility of the model simplification operation, the properties of the fillet such as radius is added as an attribute to the topological entity which is introduced in place of that fillet. In this approach, the recognized fillets are selected interactively by the user for further application of simplification operations.

The approach described above is general enough to handle various topological configurations (ring and disc type) of fillets and rounds. Also, the suppressed fillets and rounds can be reinstated. The approach, however, only deals with the constant radius blends and needs to be extended to handle variable radius fillets which are not uncommon in mechanical parts. The trace faces are identified by rule based approaches by exhaustively characterizing all the topological entities (convexity checks), which can be computationally challenging when the model size is large.

Venkataraman and Sohoni discussed simplification of blend type of features by recognizing the topological entities representing the blends and then removing the entities [Venk02b]. Tautges discussed model simplification for finite element mesh generation using hydraulic diameter based feature size metric and recognized blends and bridge features for removal from input B-Rep model [Taut01].

5.3 Arbitrary Shaped Feature Simplification

Venkatraman and Sohoni implemented the “delete face” operation to remove a set of faces corresponding to a particular feature [Venk02a]. Once the feature is removed by deleting the faces, the gaps produced are filled up by extending or contracting the neighboring faces of the removed feature. In this case, both the additive and subtractive features (in terms of volume) are considered as sets of faces. It is assumed by the authors that once the faces corresponding to the feature are removed from the model, it is possible to patch up the gaps by just contracting or extracting the neighboring faces without adding new faces. The faces that constitute a particular feature are called *feature faces*. The faces that are neighboring to the feature faces i.e. which share at least one edge or one vertex with the feature faces are called external faces. The edges that are along the boundary of the feature faces are termed as *boundary edges*, and vertices along the boundary are termed as *boundary vertices*. The edges that are not along the boundary but touch the boundary vertices are called *external edges*. Each external edge has two faces sharing it, termed as the left and the right face. When a feature is suppressed, the feature faces are removed and external edges are extended to a face known as *opposite face*. The opposite face is detected by finding the minimum Euclidean distance between the external edge and each external face. The vertex sense is another important attribute that is required to be determined for deleting the face set. The vertex sense is related to the sense of Gaussian curvature as positive or negative on the edge. The point where the external edge intersects the opposite face is called *opposite vertex*. The vertex sense of opposite vertex is determined by finding the sign of Gaussian curvature. The features suppressed using the above approach are pockets and slots. There is no error measure defined to select the features for simplification and the features are selected interactively by the user.

The technique described above is useful in suppressing both protrusion and depression types of features. The algorithm was tested on a large number of cases such as multiple boundary loops and degeneracies and found to be successful. Operations like extension, contraction and merging of neighboring faces to suppress a feature are handled effectively. The assumption that operations on neighboring faces suffice to construct the feature volume without requiring additional faces may not cater to cases where new faces need to be created for constructing feature volumes. Such problems are usually under-constrained with multiple solutions. Authors mention that additional heuristics would be required to deal with such cases. Another limitation is lack of model simplification history storage and processing and, thus, in this system, the simplified features cannot be reinstated.

The features suppressed in the approach used by Joshi are holes, fillets and bosses from a B-Rep surface model representing sheet metal components [Josh03]. The two steps followed in this paper are recognizing the feature faces and replacing them with new surface(s) to suppress the feature. The hole is recognized by determining the closed loops formed by free edges. The free edges are those which are not encountered more than once when each surface patch is traversed along their respective edges. The loop with the largest perimeter is identified as the outer boundary, while other loops as regarded as holes. Once the holes are recognized, they are suppressed by introducing a surface patch between the recognized hole's boundary or just by removing the boundary edge constraints from the NURBS representation of the surface containing the hole.

The fillets are identified by determining the curvature of iso-parameter curves at various points on the surface patch. If the curvature of all such iso-parameter curves is same, then the surface is identified as a constant radius fillet; else it is characterized as a variable radius fillet. Spring and cross edges are then identified by comparing the curvatures along the edge and perpendicular to the edge for the face under consideration and its adjacent face sharing the same edge. If the curvature along the edge is equal on both faces and corresponds to the radius of the fillet, then the edge is identified as a cross edge. If the curvature in a direction normal to the edge is greater than a threshold and also greater than the curvature of the adjacent face in the perpendicular direction, the edge is classified as a spring edge. The threshold is determined based on the bounding box of the object. After this, the edges of the fillet are chained and sequenced. Once the fillets are recognized and sequenced, they are suppressed as follows. The edges of the fillet surface that are common with the support surface are extended in tangential direction of the fillet and the intersection curve is determined. Thus, the fillet surface is replaced with two new planar surfaces sharing edges with the support surface and the newly generated intersection curve.

The boss is recognized after the fillets are already recognized and fillet chains and rings are identified. The algorithm assigns an attribute to each face specifying the fillet chains and rings contained in it. For all planar faces the adjacent faces on its outermost loop are checked and if they belong to the same fillet ring, the planar face is marked as “boss top” and the fillet ring is marked as “top ring” face. After this, adjacent faces of the top ring are checked to identify the bottom ring. All these surface patches are recognized as parts of the boss. To suppress the boss all the components of the boss like the top face, the top ring and the bottom ring are removed and replaced by a smooth continuous flat surface patch. To do this, the edges between the bottom ring and base surface are chained to form a closed loop and a flat surface patch is created for this loop in the same way as it is done for a hole. The generated edge may not be the one that existed before the blend generation. The reason for this is that the effect of curvature of freeform base surface is not accounted for while calculating the intersection curve. This approach can sometimes recognize a surface as a blend even if the surface is not intended to be so.

The concept described above is applied for mesh generation for Finite Element Analysis, particularly for design of forming dies and molds where small holes and bosses are not very important in the early stages of analysis. The quality of mesh is analyzed with respect to the number of elements, aspect ratio, warping and the mean value of element included angle. It is shown that for several test cases, the mesh quality for the given criterion improves after feature simplification. The approach is particularly advantageous in handling freeform surfaces. The method can tackle arbitrary shaped holes and variable radius fillets and rounds. Compound features such as darts and beads are reportedly handled by this approach which is useful for a completely new family of parts having such features. The features cannot be recovered once they are removed from the model database as no simplification history is maintained. Moreover, maintaining a history of operation and reinstatement of feature may be difficult to incorporate in this architecture.

Kim *et al.* reported a system for multi-resolution feature simplification using three operators, namely wrap-around, smooth out and thinning [Kim05]. The method is applied to finite element model preparation and network transmission multi-resolution modeling. The features are first recognized using rule based techniques and then various operators are applied to suppress them. In case of wraparound operator, the model is considered to be wrapped by a thin plastic cover. The parts of model hidden by the cover are considered for simplification or removal. The wraparound operators are explained in the earlier papers by Koo and Lee [Koo02] and Seo *et al.* [Seo05]. This operator is especially suitable for handling concave features. The operator is applied to features that are detected as blends, chamfers and concave features. Authors have applied certain rules to recognize feature. For example, the existence of cylindrical surface on concave edge is recognized as a fillet and the existence of inner loops of convex edges connected to more than one face is recognized as a concave feature. The recognized features are then simplified using wrap-around operator. The limitation of wrap-around operator is its volume-additive nature in case of a protrusion type of feature. If the protrusion feature is very small, a volume removal approach is more useful.

To address convex features such as bosses and ribs, authors propose smooth out operator, which can be considered as smoothing operation carried out by a sand paper or chisel. The bosses are identified by finding concave inner loops connected to more than one face. The ribs are identified by finding face pairs which are much smaller in size than the other faces and contain two concave edges. If two faces are connected by a convex edge, then the pair is identified as a rib face set. Once the features are identified, the smooth-out operator removes the faces corresponding to each boss and rib and the neighboring faces are extended and stitched to fill the gaps produced by removal of feature. Smooth out operator cannot be applied to the concave features. Finally, thinning operator is applied to model already simplified by wrap-around and smooth out operators. In the thinning process, the dimension of features is reduced to 2D or 1D using mid-surface representation. The model is searched for face pairs having same geometry (the geometries supported are plane, cylinder, sphere, cone, torus, and offset face). If the distances between pairs of faces are found to be very less as compared to the face dimensions, the pairs are identified as candidate for dimension reduction using thinning operators. The thinning operator introduces an additional face between the faces in the pairs recognized for thinning and the introduced face is trimmed to the shape of bounding box of the face pair. After this, the thickness information is attached to the introduced face and it is stitched to the model. In each of the operators explained, the history of simplification is stored and the reinstatement of the simplified features in each case can be obtained using the history information.

The major advantages of the method described above are its applicability to assembly models and interoperability. The implementation is developed in Parasolid kernel (used in many commercial CAD systems) and hence the algorithm is portable. The rule based feature recognition is one of the computational overheads and authors propose that feature based models can be used to overcome this limitation. In case of simplification of assembly models, the interface between mating parts is not considered and hence the mating information is not preserved.

All the techniques mentioned in this section are summarized in Table 3.

6. Techniques based on Dimension Reduction

In many applications, reducing the dimensions of CAD models is beneficial. For example, consider a long round slender bar of uniform diameter. If we model this long slender round bar as a 1D beam, there will be negligible effect on the accuracy of the analysis applied to the beam but the computational time will reduce dramatically. Hence, dimensional reduction is studied by the physics-based simulation community (especially finite element analysts) for model simplification purposes.

6.1 Medial Axis Transform based Dimension Reduction

One of the well established techniques for dimension reduction is medial axis transform (MAT). There is a large body of literature on MAT and it is difficult to include all of them here. We, thus, suggest the interested readers to refer to the survey paper by Attali *et al.* for details about medial axis transform based techniques [Atta04]. In this section, we will discuss few representative papers that use MAT based model simplification. In addition we will also discuss techniques based on θ-MAT which is a modified approach for MAT and mid-surface abstraction.

Donaghy *et al.* used Geometry Idealization pertaining to dimension reduction to simplify a model for analysis and other downstream operations [Dona00]. The technique used for dimension reduction used by authors is Medial Axis Transform (MAT). The process of development of MAT follows three steps. In the first step, Delaunay triangulation of the object is performed. In the second step, the circum-circles for each triangle generated are determined. In the last step, the circum-centers are fitted on a curve. The fitted curve is called the medial axis.

To determine whether the dimension of the object is to be reduced, aspect ratio and taper criteria are used. The lower bound for aspect ratio is determined as the ratio of the length of the shortest edge bounding the region and the maximum disk diameter in the local region. The taper is determined as the maximum rate of change of diameter with respect to medial edge length. The high aspect ratio or low threshold indicates variation in slenderness property of the object. If an object region has an aspect ratio greater than the threshold or a taper value lower than the taper threshold, then the region is suitable for modeling with 1D element. If not, then the region is either kept in its original dimension for meshing with 2D elements, or dimensionally reduced to an equivalent 0D-point element.

After dimensional reduction has been performed, physical properties are imparted to the reduced dimension model. The main parameters related to physical properties discussed by the authors in the context of FEA are the moment of inertia tensor and the position of the neutral axis. This approach is used for dimension reduction of 2D planar or 3D shell model. The technique is not applicable for simplification of other types of 3D solids.

Sud *et al.* presented an algorithm for homotopy preserving medial axis simplification of polyhederal models with a linear computational complexity and proposed applications to mesh generation and shape analysis [Sud05]. Fosky *et al.* utilized the concept of θ-MAT for dimension reduction and model simplification of polyhederal mesh models [Fosk03]. θ-MAT is more stable algorithm compared to MAT computationally. The potential applications of the approach suggested by Foskey *et al.* are in the area of mesh generation and shape analysis.

6.2 Mid-surface Abstraction

Rezayet presents a technique to abstract the part model in terms of mid-surface [Reza98]. The main applications of this approach are in FEA model preparation and feature recognition. Authors have pointed out several benefits of mid-surface abstraction in comparison with medial axis transform such as volume preservation, non-creation of MAT branches, effective simplification of features and detail removal, reflection of part form and use of geometric reasoning to define the shape. There are four steps involved in the generation of mid-surface, namely pairing surfaces, topology based adjacency graph creation, mid-surface patch generation and sewing the patches based on adjacency information.

In the surface pairing step, all the faces excepting the end-caps (faces on the edges of solid model) and orphans (faces not on the thin wall sections) are paired. The pairing process involves arbitrary selection of a face as the seed face and casting of a ray in the material direction from the seed face. If the ray hits another face, the faces are paired and all the related faces i.e. the faces sharing common edges with the paired faces are tagged and processed. From the remaining faces, again a seed face is selected and the process is repeated till all the faces are processed. The processed faces are then used to create adjacency graph. The nodes of the graph are the faces and the arcs are the relationships between the faces. The two types of relationships defined are pairing relationship and the common edge relationship. The patterns in the graph indicate particular types of features.

The paired faces are then interpolated to create mid-surface patches, which may be intersecting. The topological adjacency information is used to trim the patches. In case of plane faces, a 2 x 2 grid is used, while in case of non-plane faces, a 15 x 15 grid is used to create the interpolated points. The thickness distribution is then assigned to the mid-surface thus generated. The thickness of the part at an arbitrary point P, on the mid-surface between two parent surfaces S_1, and S_2 is T, where $T = dist(P, S_2) + dist(P, S_1)$. (6)

The mid-surface abstraction is applied to the entire model simultaneously and no error threshold is defined to select the parts of the model to which abstraction is to be applied. In this approach, small features such as holes are simplified automatically when the mid-surface is generated. Material distribution is effectively represented in this approach by introducing thickening in the direction of the draft and by introducing thinning in the direction of the undercut.

In the area of dimension reduction, other reported work is by Chong *et al.*, who presented a technique to decompose the solid model into parts and then applied mid-surface abstraction for simplification for finite element model preparation application [Chon04].

All the techniques listed in this section are summarized in Table 4.

7. Discussion

We studied existing model simplification techniques that are useful from physics based simulation point of view and classified them broadly into four main categories based upon the type of simplification operators used in the respective techniques, *i.e.* surface entity, volumetric entity, explicit feature and dimensional reduction. Fig. 9 shows the taxonomy of different model simplification techniques in the form of a tree containing four hierarchical levels. The solid lines denote direct inheritances of different techniques from parent classes.

Analysis of the four Tables presented in the preceding Sections clearly reveals that there is no single technique that can solve all the model simplification problems. For example, a majority of the techniques are applicable for FEA model preparation, whereas others are suitable for fluid flow problems, collision detection or even as pre-processing steps in recognizing complex features. Again, some of the techniques can effectively handle prismatic features, while others are useful in dealing with shell-based, freeform, cavity or protrusion type of features. The level of automation, input model format and the type of operators used vary quite a bit as well. All these factors, namely, application domain, types of features handled, input format, level of automation and type of operators used, need to be taken into consideration before selecting a particular technique. Table 5 has been drawn to summarize our findings. We believe that this will aid potential users in choosing the appropriate technique quickly based upon the characteristics of their problem.

8. Conclusions

This paper summarizes various model simplification techniques available in the open literature that are applicable for physics-based simulation applications. To the best of our knowledge, this is the first attempt towards classifying and organizing the various techniques into well-defined categories. Comparative study of all the techniques in each of the categories has been performed in order to delineate the types of features handled, relative merits, weaknesses, and potential applications clearly. Based on this classification and comparative study, we have also presented a broad selection criterion for different classes of engineering problems commonly encountered in practice.

This literature survey clearly reveals that there are many open research issues that merit serious attention in the future. They are listed as follows:

- *Lack of formal analysis of computational complexity* – It is possible to identify certain methods to be computationally less intensive than others based on computational experiments on test data. However, very few of them have formally enumerated the complexity in terms of input parameters. In this case, the overall complexity can be decomposed into two categories: one pertaining to the recognition or identification of features and the other pertaining to the actual simplification. The former will be associated with the number of vertices or faces based on the type of solid model used (e.g. mesh-based methods will relate it to vertices, whereas B-Rep-based methods will link it to the number of faces or surface patches). The latter on the other hand will deal directly with the number of features implicitly or explicitly represented in a particular method. Thus, formal derivation of asymptotic computational complexity is often a complex task. However, to really understand the nature of the underlying algorithms and make further progress, it will be very useful to know the asymptotic time complexity of the algorithms.
- *Lack of application-specific (physics-dependent) error measures* – A vast majority of methods utilizes indirect, geometry based error metrics to characterize their performance. These errors are used as thresholds to arrive at various decisions regarding classification and consequent simplification of individual features. However, these metrics usually cannot directly address the error that will be introduced in terms of the physical behavior and properties of the system under various possible external loading conditions. Hence, such metrics need to be developed using a mathematically rigorous framework to estimate the errors accurately and efficiently.
- *Lack of standardized set of test parts* – Graphics community has developed several standard test cases to assess the performance of any newly developed simplification technique for rendering (e.g., Stanford bunny, sculpture of David by Michelangelo [Stan07]). However, current the physics-based simulation community lacks a standard set of test parts to test simplification algorithm performance. So the research community needs to come up with a basic, test set of solid models that is acceptable to everyone working in this field.
- *Lack of formal investigation of robustness* – Robustness is an important issue in geometry computing applications. Discrepancies in floating point representation of geometric entities may result in erroneous results. Hence, often significant effort is devoted to ensuring robustness of the proposed algorithms. The problem of robustness has not yet received significant attention in model simplification applications.

References

[Andu02] Andújar, C., Brunet, P., Ayala, D., (2002). "Topology reducing surface simplification using discrete solid representation." *ACM Transactions on Graphics*. 21(2):88-105.

[Arms94] Armstrong, C., G., "Modelling requirements for finite-element analysis." *Computer-Aided Design.* 26(7):573-578.

[Atta04] Attali, D., Boissonnat, J., D., Edelsbrunner, H., (2004). “Stability and computation of the medial axis – a state-of-the-art report.” *Mathematical Foundations of Scientific Visualization, Computer Graphics, and Massive Data Exploration. Springer-Verlag. Berlin.*

[Bida93] Bidarra, R., Teixeira, J., C., (1993). “Intelligent form feature interaction management in a cellular modeling scheme.” *2nd ACM Solid Modeling Montreal Canada.*

[Bida98] Bidarra, R., deKraker, R., K., Bronsvoort, W., (1998). “Representation and management of feature information in a cellular model.” *Computer-Aided Design.* 30(4):301–313.

[Bida05] Bidarra, R., Madeira, J., Neels, W., J., Bronsvoort, W., F., (2005). “Efficiency of boundary evaluation for a cellular model.” *Computer-Aided Design*. 37(12):1266-1284.

[Blum67] Blum, H., (1967). “A transformation for extracting new descriptors of shape.” *Analysis and Machine Intelligence.* 17(1). 635–640.

[CADf05] CADfix (2005). http://www.cadfix.com, ITI TranscenData, Interoperability Solutions for CAD/CAM/CAE.

[Cava97] Cavaleanti, P., R., Carvalho., P., C., P., Martha, L., F., (1997). “Non-manifold modeling: an approach based on spatial subdivision.” *Computer-Aided Design*. 29(3):209-220.

[Chen93] Chen, J., Gürsöz, E., L., Prinz, F., B., (1993). “Integration of parametric geometry and non-manifold topology in geometric modeling.” *Proceedings on the second ACM symposium on Solid modeling and applications. New York. NY.*

[Choi02] Choi, D., H., Kim. T., W., Lee, K., (2002). “Multiresolutional representation of B-*Rep* model using feature conversion.” *Transactions of the Society of CAD/CAM Engineers.* 7(2):121-130.

[Chon04] Chong, C., S., Kumar, A., S., Lee, K., H., (2004). “Automatic solid decomposition and reduction for non-manifold geometric model generation.” *Computer-Aided Design*. 36(13):1357-1369.

[Cign98] Cignoni, P., Montani, C., Scopigno, R., (1998). “A comparison of mesh simplification algorithms.” *Computers & Graphics*. 22(1):37-54.

[Dabk94] Dabke, P., Prabhakar, V., Sheppard, S., (1994). “Using Features to Support Finite Element Idealizations.”*Proceedings of the ASME Conference on Computers in Engineering*. Minneapolis. MN.

[Date05] Date, H., Kanai, S., Kisinami, T., Nishigaki, I., Dohi, T., (2005). “High-Quality and Property Controlled Finite Element Mesh Generation From Triangular Meshes using the Multiresolution Technique.” *Journal of Computing and Information Science in Engineering.* 5(4):266-276.

[Date06] Date, H., Kanai, S., Kishinami, T., Nishigaki, I., (2006). “Flexible feature and resolution control of triangular meshes.” *Proceedings of the Sixth IASTED International Conference on Visualization, Imaging and Image Processing. Palma de Mallorc. Spain.*

[Dey97] Dey, S., Shepard., M., S., Georges, M., K., (1997). "Elimination of the adverse effects of small model features by the local modification of automatically generated meshes." *Engineering with Computers.* 13(3):134-151.

[Dona96] Donaghy, R., J., McCune, W., Bridgett, S., J., Armstrong, C., G., Robinson, D., J., Mckeag, R., M., (1996). "Dimensional reduction of analysis models." *Proceeding of 5th International Meshing Roundtable. Pittsburgh. PA.*

[Dona00] Donaghy, R., J., Armstrong, C., G., Price, M., A., (2000). "Dimensional reduction of surface models for analysis." *Engineering with Computers*. 16 (1):24-35.

[Elsa98] El-Sana, J., Varshney, A., (1998). "Topology simplification for polygonal virtual environments." *IEEE Transactions on Visualization and Computer Graphics*. 4(2) :133-144.

[Foca08] Foucault, G., Cuillière, J., François, V., Léon, J., (2008). "Adaptation of CAD model topology for finite element analysis." *Computer-Aided Design.* 40(2):176-196.

[Fine00] Fine, L., Remondini, L., Leon, J. C., (2000). "Automated generation of FEA models through idealization operators." *International Journal for Numerical Methods in Engineering*. 49(1-2):83-108.

[Fosk03] Foskey, M., Lin, M., C., Manocha, D., (2003). "Efficient computation of a simplified medial axis." *Journal of Computing And Information Science In Engineering.* 3(4):274-284.

[Garl97] Garland, M., Heckbert, P., (1997). "Surface simplification using quadric error metrics." *Proceedings of ACM SIGGRAPH '97. Los Angeles. CA.*

[Gome93] Gomes, A., Bidarra, R. and Teixeira, J., C., (1993). "A cellular approach for feature-based modeling." *In Graphics Model@ and Visualization in Science and Technology. Ed. M. Gobel and J. C. Teixeira. Springer-Verlag. Heidelberg.* 128- 143.

[He95] He, T., Hong, L., Kaufman, A., Varshney, A., Wang, S., (1995). "Voxel based object simplification." *Proceedings of Visualization'95. Atlanta. GA.*

[Hoff89] Hoffmann, C., M., (1989). "Geometric and solid modeling: an introduction." *Morgan Kaufmann Publishers Inc. San Francisco. CA.*

[Inou01] Inouea, K., Itoha, T., Yamadaa, A., Furuhatab, T., Shimadac, K., (2001). "Face clustering of a large-scale CAD model for surface mesh generation." *Computer-Aided Design*. 33(3):251-261.

[Josh03] Joshi, N., Dutta, D., (2003). "Feature simplification techniques for freeform surface models." *Journal of Computing and Information Science in Engineering.* 3:177-186.

[Kim05] Kim, S., Lee, K., Hong, T., Kim M., Jung, M., and Song, Y. (2005). "An integrated approach to realize multi-resolution of B-Rep model." *Proceedings of the 2005 ACM Symposium on Solid and Physical Modeling. Cambridge. MA.*

[Koo02] Koo, S., Lee, K., (2002). "Wrap-around operation to make multi-resolution model of part and assembly." *Computers & Graphics.* 26(5):687-700.

[Lee98] Lee, Y., G., Lee, K., (1998). "Geometric detail suppression by the Fourier transform."

Computer-Aided Design. 30(9): 677-693.

[Lee99] Lee, K., (1999). "Principle of CAD/CAM/CAE system." *Addison-Wesley/Longman. Reading.*

[Lee01] Lee, S., H., Lee, K., (2001). "Partial entity structure: A compact boundary representation for non-manifold geometric modeling." *Journal of Computing and Information Science in Engineering.* 1(4):356-365.

[Lee02] Lee, S., H., Lee, K., S., Park, S., (2002). "Multiresolution representation of solid models using the selective Boolean operations." *Proceedings of the 2002 Spring Conference of Korean Society of Precision Engineering. Daejeon. Korea. (In Korean)*

[Lee03] Lee, K., Y., Price, M., A., Armstrong, C., G., Larson, M., Samuelsson, K., (2003). "CAD-TO-CAE integration through automated model simplification and adaptive modeling." *Proceedings of International Conference on Adaptive Modeling and Simulation. Barcelona. Spain*.

[Lee04] Lee, J., Y., Lee, J., H., Kim, H., Kim, H., S., (2004). "A Cellular topology-based approach to generating progressive solid models from feature-centric models." *Computer-Aided Design.* 36(3):217-229.

[Lee05a] Lee, K., Y., Armstrong, C., G., Price, M., A., Lamont J., H., (2005). "A small feature suppression/unsuppression system for preparing B-*Rep* models for analysis." *Proceedings of the 2005 ACM symposium on Solid and physical modeling. Cambridge. MA.*

[Lee05b] Lee, S., H., (2005). "A CAD-CAE integration approach using feature-based multi-resolution and multi-abstraction modeling techniques." *Computer-Aided Design.* 37(9):941-955.

[Lee05c] Lee, S., H., (2005). "Feature-based multiresolution modeling of solids." *ACM Transactions on Graphics.* 24(4):1417-1441.

[Lee05d] Lee, S., H., Lee, K., Lee, K., Y., (2005). "Feature-based multiresolution and multi-abstraction non-manifold modeling system to provide integrated environment for design and analysis of injection molding products." *Proceedings of the First Korea-China Joint Conference on Geometric and Visual Computing. Busan. Korea.*

[Lee06a] Lee, S. H., Lee, K., (2006). "Feature-based multiresolution techniques for product design." *Journal of Zhejiang University SCIENCE A.* 7(9):1535-1543.

[Lee06b] Lee, S., H., Lee, K., Kim, S., C., (2006). "History-based selective □tanfor operations for feature-based multi-resolution modeling." *Proceedings of ICCSA 2006. Glasgow. United Kingdom.*

[Li02] Li, B., Liu., J., (2002). "Detail feature recognition and decomposition in solid model." *Computer-Aided Design*. 34(5):405-414.

[Lim04] Lim, T., Medellin, H., Corney, J., R., Ritchie, J., M., Davies, J., B., C., (2004). "Decomposition of complex models for manufacturing." *Proceedings of the Shape Modeling and Applications. Cambridge. MA.*

[Lueb01] Luebke D., P., (2001). "A developer's survey of polygonal simplification algorithms." *IEEE Computer Graphics Applications*. 21(3):24-35.

[Lueb02] Luebke, D., P., Watson, B., Cohen, J., D., Reddy, M., Varshney, A., (2002). "Level of Detail for 3D Graphics." *Elsevier Science Inc. New York. NY.*

[Mast08] Masters in the art of airbrush automotive illustration**.** http://www.khulsey.com/masters_makoto_ouchi.html.

[Masu93] Masuda, H., (1993). "Topological operators and Boolean operations for complex-based non-manifold geometric models." *Computer-Aided Design*. 25(2):119-129.

[Mobl98] Mobley, A., V., Carroll, M., P., Canann, S., A. (1998). "An object oriented approach to geometry defeaturing for finite element meshing." *Proceedings of 7th International Meshing Roundtable. Dearborn. MI.*

[Ragh98] Raghothama, S., Shapiro, V., (1998). "Boundary representation deformation in parametric solid modeling." *ACM Transactions on Computer Graphics*. 17(4):259-286.

[Rama05] Ramanathan, M., B. Gurumoorthy, B., (2005). "Constructing medial axis transform of extruded and revolved 3D objects with free-form boundaries." *Computer-Aided Design.* 37(13): 1370-1387.

[Reza96] Rezayat, M., (1998). "Midsurface abstraction from 3D solid models: general theory and applications." *Computer-Aided Design*. 28(11):905-915.

[Ribe01] Ribelles, K., Heckbert, P., S., Garland, M., Stahovich, T., Shivastava, V., (2001). "Finding and removing features from polyhedra." *Proceedings of ASME DETC'01. Pittsburgh. PA.*

[Robi06] Robinson, T., T., Armstrong, C., G., McSparron, G., Quenardel, A., Ou, H., McKeag, R., M., (2006). "Automated mixed dimensional modeling for the finite element analysis of swept and revolved CAD features." *Proceedings of the 2006 ACM symposium on Solid and physical modeling. Cardiff. Wales. United Kingdom.*

[Saad03] Saad, Y., (2003). "Iterative methods for sparse linear systems." *Society for Industrial and Applied Mathematics. Philadelphia. PA.*

[Seo05] Seo, J., Song, Y., Kim, S., Lee, K., Choi, Y., Chae, S., (2005). "Wrap-around operation for multi-resolution CAD model." *Computer-Aided Design & Applications.* 2(1-4): 67-76.

[Shah94] Shah, J., J., Mantyla, M., Nau, D., S., (1994). "Advances in feature-based manufacturing." *Elsevier Science Inc. New York. NY.*

[Shah95] Shah, J., J., Mäntylä, M., (1995). "Parametric and feature-based CAD/CAM." *Wiley. New York. NY.*

[Shef97] Sheffer, A., Blacker, T., Bercovier, M. (1997). "Clustering: automated detail suppression using virtual topology." *AMD 220. Trends in Unstructured Mesh Generation. ASME. Evanston. IL.*

[Shef01] Sheffer, A., (2001). "Model simplification for meshing using face clustering." *Computer-Aided Design*. 33(13):925-934.

[Shep97] Dey, S., Shepard., M., S., Georges, M., K., (1997). "Elimination of the adverse effects of small model features by the local modification of automatically generated meshes." *Engineering with Computers.* 13(3):134-151.

[Shep98] Shephard, M., S., Beall, M., W., O''Bara, R., M., (1998). "Revisiting the elimination of the adverse effects of small model features in automatically generated meshes." *Proceedings of 7th International Meshing Roundtable. Dearborn. MI.*

[Stan07] Stanford 3D Scanning Repository (2007). http://graphics.stanford.edu/data/3Dscanrep

[Sud05] Sud, A., Foskey, M., Manocha, D., (2005). "Homotopy-preserving medial axis simplification." *Proceedings of the 2005 ACM symposium on Solid and physical modeling. Cambridge. MA.*

[Tan99] Tan, T., Low, K., (1999). "Computing bounding volume hierarchies using model simplification." *Proceedings of Symposium on Interactive 3D Graphics. Atlanta. GA.*

[Taut01] Tautges, T., J., (2001). "Automatic detail reduction for mesh generation applications." *Proceedings of the 10th International Meshing Roundtable. Newport Beach. CA.*

[Venk02a] Venkataraman S., Sohoni M., (2002). "Reconstruction of feature volumes and feature suppression." *Proceedings of 7th ACM Symposium on Solid Modeling 2002. Saarbrucken. Germany.*

[Venk02b] Venkataraman, S., Sohoni, M., Rajadhyaksha, R., (2002). "Removal of blends from boundary representation models." *Proceedings of 7th ACM Symposium on Solid Modeling 2002. Saarbrucken. Germany.*

[Vero98] Veron P., Leon J., C., (1998). "Shape preserving polyhedral simplification with bounded error." *Computers & Graphics.* 22(5):565-585.

[Whit03] White, D. R., Saigal, S., Owen, S. J. (2003). "Meshing complexity of single part CAD models." *Proceedings of the 12th International Meshing Roundtable Conference. Santa Fe. NM.*

[Yoon04] Yoon, S., Salomon, B., Lin, M., Manocha, D., (2004). "Fast collision detection between massive models using dynamic model simplification." *Proceedings of Eurographics Symposium on Geometry Processing. Nice. France.*

[Zhu02] Zhu, H., Menq C., H., (2002). "B-*Rep* model simplification by automatic fillet/round suppressing for efficient automatic feature recognition." *Computer-Aided Design*. 34(2):109-123.

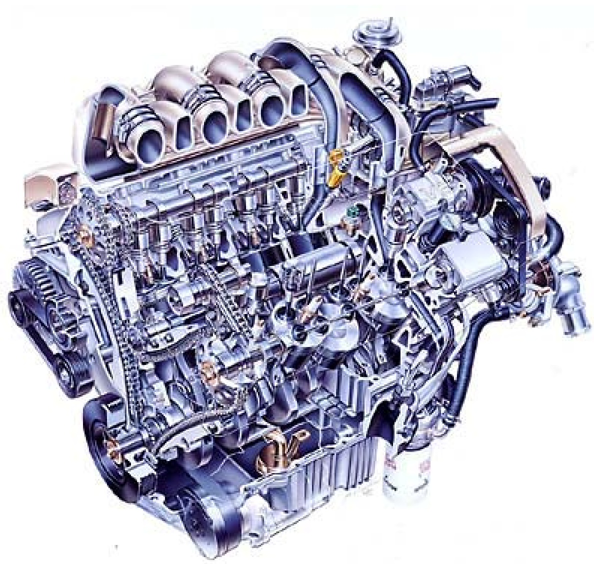

Fig.1. A 32 Valve SOHC V8 Engine [Mast08]

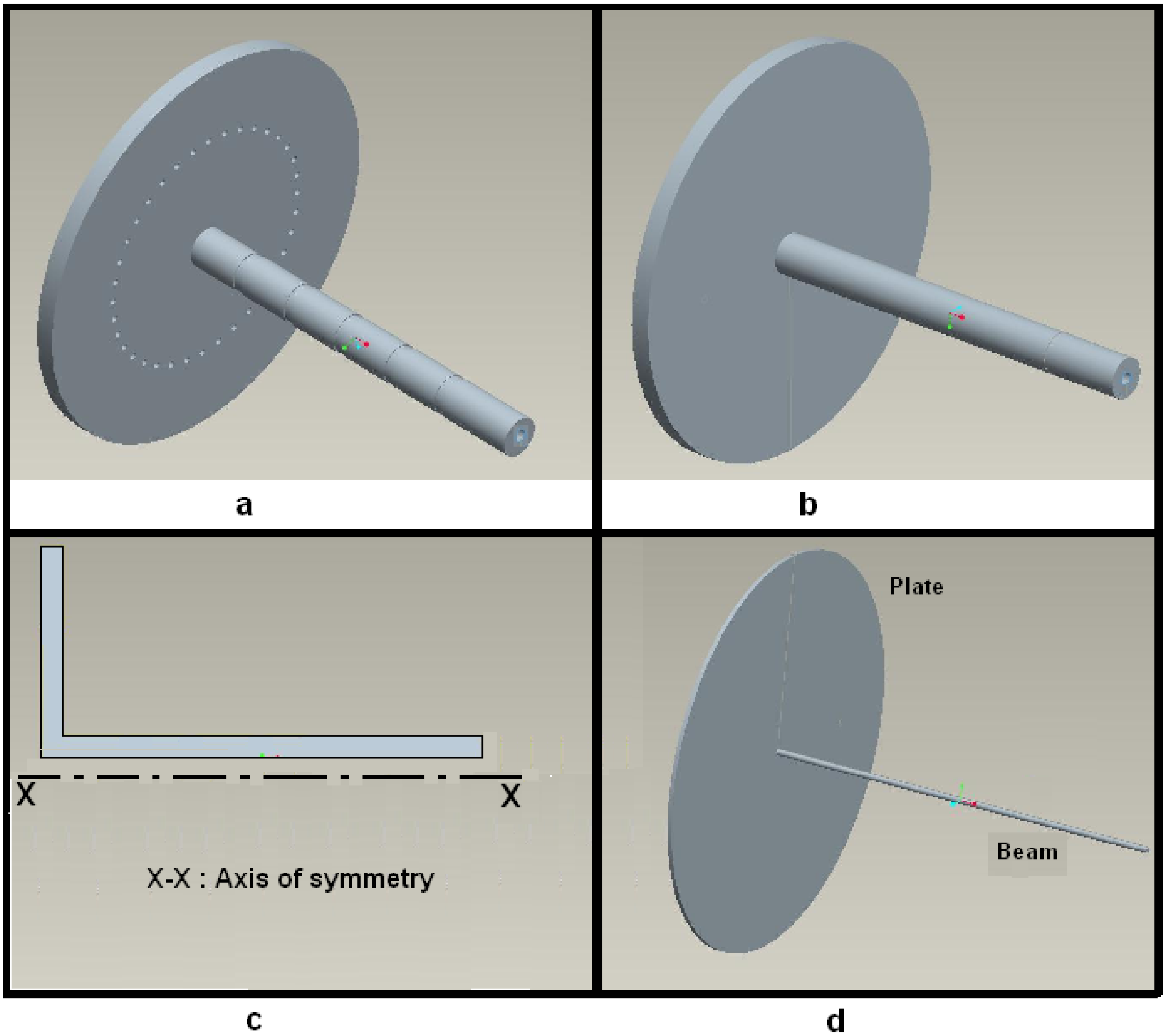

Fig. 2. Model simplification example; (a) 3D Model, (b) Simplified model after removing notches and tiny holes, (c) Simplified model after dimension reduction and exploiting symmetry, (d) Simplified model after dimensional reduction to combination of beam and plate elements

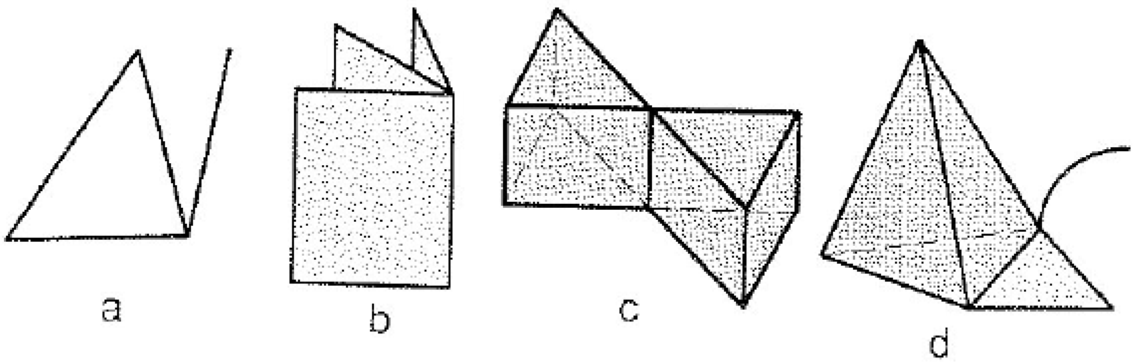

Fig. 3. Some examples of non manifold cell complexes [Masu93]

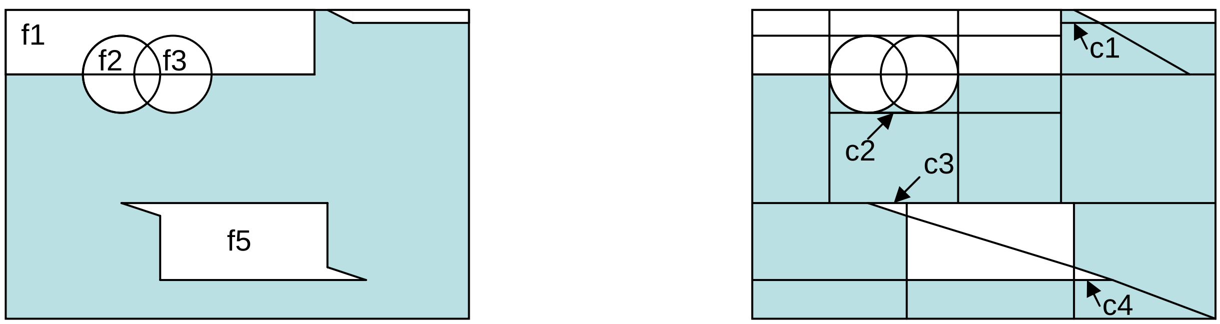

Fig. 4. An example of cellular decomposition: (a) A part created by removing five subtractive feature from a block; (b) Cellular model consisting of 35 cells created by feature volume splitting and controlled half space partitioning

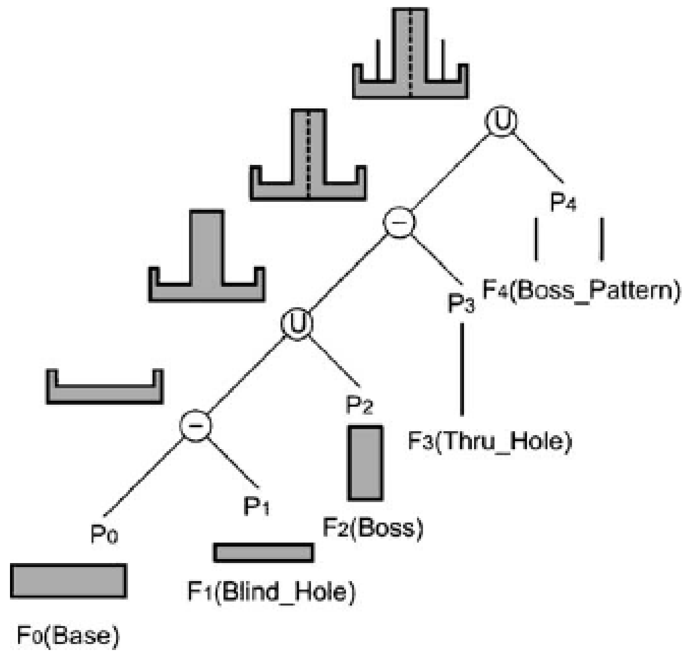

Fig. 5. An example of NMT Modeling [Lee05b]

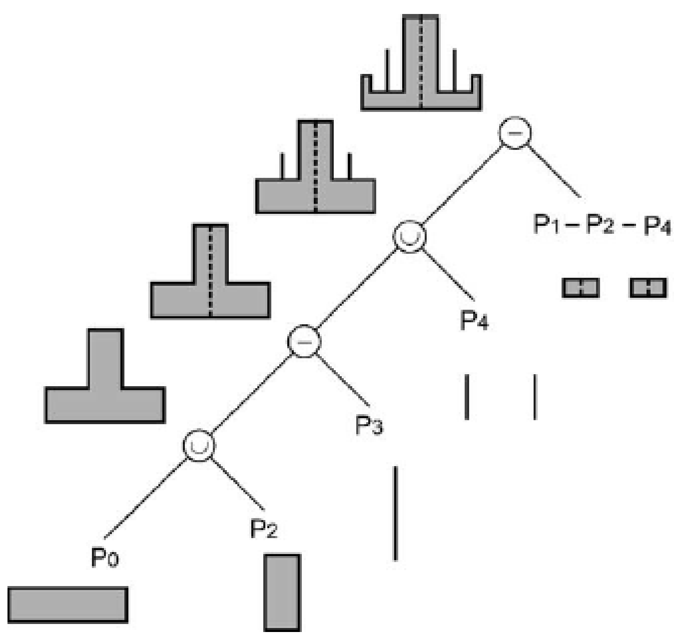

Fig. 6. Feature rearrangement based on "effective zone of feature" [Lee05b]

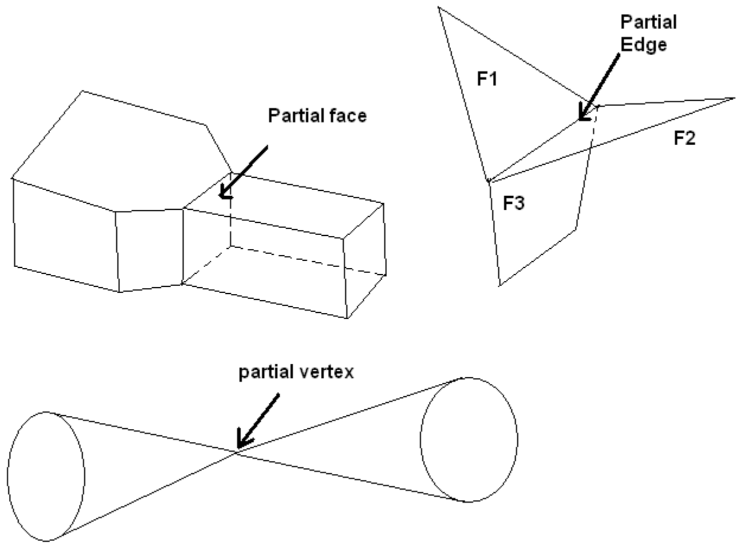

Fig. 7. Partial Entities

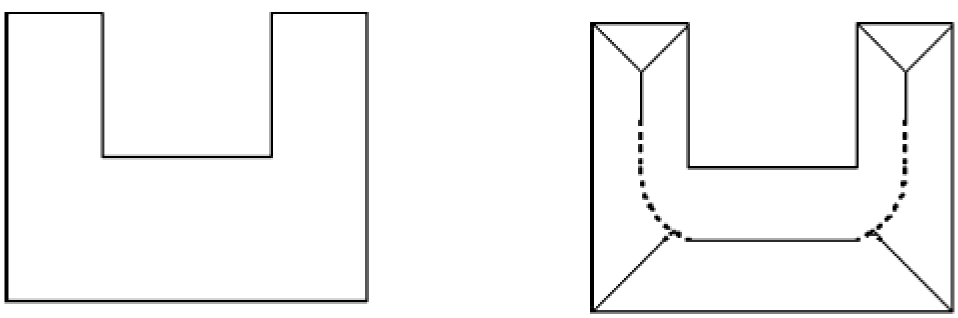

Fig. 8. An example of medial axis transform: (a) A channel section; (b) Medial axis transform of the channel section [Rama05]

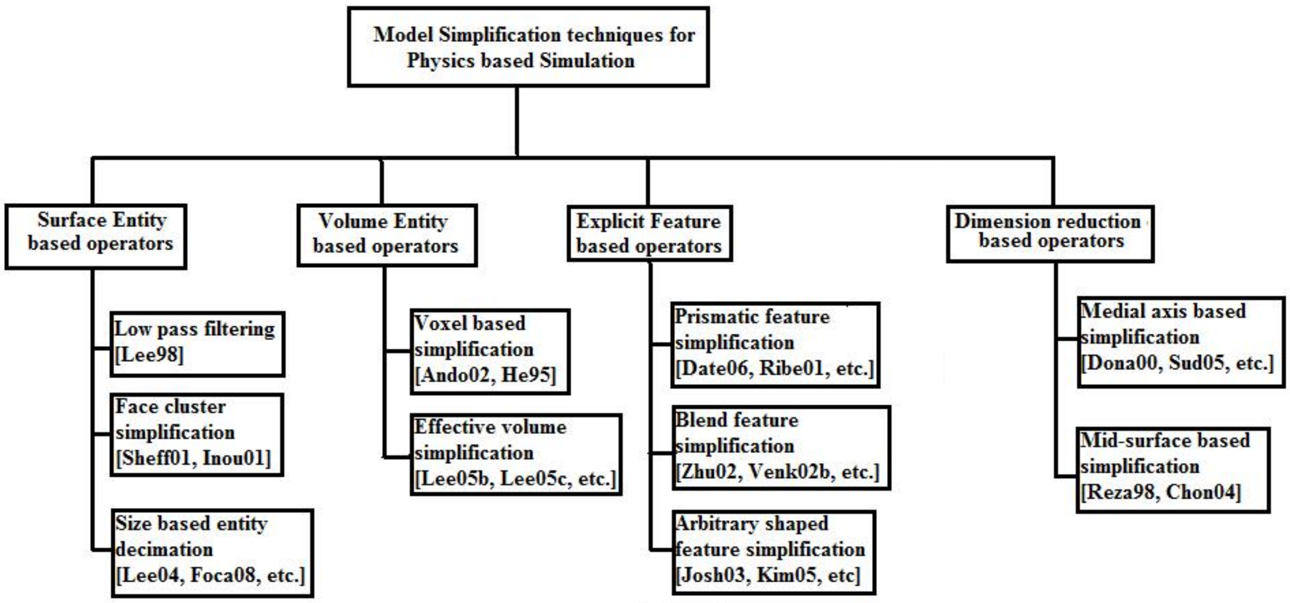

Fig. 9. Taxonomy of model simplification techniques

Table 1: Summary of techniques based on surface entity based operators for model simplification

Method	Input format	Features simplified	Simplification criterion	Advantages	Limitations	Application domain
Fourier Transform based low pass filtering [Lee98]	2D image	Small boundary edges and island type features	Based on error measure computed as average distance between original and LPF models	Error metric based LOD selection reduces computation time; can be used in feature recognition	Lack of automation; applicable to 2D surfaces; performance issues in 3D extension	FEM model preparation
Face clustering [Shef01]	Gambit pre-processor (Fluent™)	Chamfers, fillets, rounds and spherical surface based features	Metric based on boundary preservation, region size, region smoothness and simplicity of region boundary shape	Applicable to both faceted and free form geometries; curved regions can be handled; geometry is retained, only topology changes	face clustering may not be optimal	FEM model preparation
Face Clustering [Inou01]	Polygonal Mesh	Protrusions and depressions	Error measure based on cluster area, cluster boundary smoothness and flatness	Mesh quality based metrics are used	Faces with large variation in normal vectors not handled; face clustering may not be optimal	Mesh generation for FEM analysis
Face Clustering [Dey97, Shep98]	Polygonal mesh	Protrusions and depression	Aspect ratio and dihederal angle of elements	Approach is simple to implement	Through hole removal may not be possible	FEM mesh generation
Decimation - Cellular topology progressive modeling [Lee04]	Feature based model	Freeform features	Based on ascending order of progressive volumes	Applicable for NURBS surfaces; computationally efficient	Application specific feature complexity not considered	Network transmission; FEM model preparation
Decimation-Cell transformation based technique [Lee05a]	B-Rep	Holes, fillets and narrow regions	The features are selected manually by the user for simplification	Analysis history useful for simplification strategy comparison; Handles narrow regions	Post processing needed for mesh generation of desired density; Lack of automation	FEM model preparation
Decimation-MCT based technique [Foca08]	B-Rep	Freeform	Interactive	Preserves topology and creates new geometry adapted to various mesh quality constraints	Lack of automation	FEM model preparation

Decimation-Idealization operators based on vertex removal and spherical error zone [Fine00]	Polygonal Mesh	Holes	Error measure based on a posteriori analysis using tetrahedral finite elements and Interactive	Takes care of large transformation s and simplification of complex areas of polyhedron such as saddle points	Only for polyhedral models; error estimator for hexahedral elements not developed	FEM model preparation
Decimation-OOP approach for defeaturing [Mobl98]	Polygonal Mesh	Loops, coincident edges, near tangencies	Error measure related to global defeaturing tolerance	Scalable object oriented software design	protuberances are not handled	FEM Model preparation
Edge decimation based approach [Date05]	Polygonal Mesh	Protrusions and depressions	Error measure based on overall geometry error, mesh size and mesh shape	Mesh quality based error metrics are used	Not applicable for assembly models	Mesh generation for FEM analysis
Decimation - Shape preserving polyhederal simplification [Vero98]	Polygonal mesh	Protrusions and depression	Shape based error zone and discrete form of Gaussian curvature	Object shape preserved	May not handle large geometric transformatio n in complex area (such as saddle points)	Mesh generation for FEM analysis

Table 2: Summary of techniques based on volumetric entity based operators for model simplification

Method	Input format	Features simplified	Simplification criterion	Advantages	Limitations	Application domain
Voxel based-Discretized Polyhedra Simplification based topology reducing surface simplification [Andú02]	Polygonal Mesh	Holes, protuber-ances and edge features	Hausdorff distance between MDCO and original model	Handles assembly models; non-manifold inputs can be handled	Pre-processing required	Collision detection, occlusion analysis, multi-resolution robust Boolean operations, indirect illumination acoustic modeling and query acceleration
Voxel based object simplification [He95]	Polygonal Mesh	Small protuberan ces and cavities	Frequency threshold for low pass filtering	Handles individual as well as collection of objects	Large number of redundant triangles generated for low surface curvature regions	Efficient antialiased Rendering; can be used for collision detection
Effective volume based technique [Lee05b, Lee05c, Lee06a, Lee06b]	Polygonal Mesh	Freeform features	Volume threshold	Capable of multi-resolution modeling	Depends on feature volume and not on complexity; when the shape or semantics of feature are altered, predefined abstract model may become invalid; physical constraints and properties applied to CAD models not transferred to CAE models	CAD-CAE integration and network model transmission

Table 3: Summary of techniques based on explicit feature simplification operators

Method	Input format	Features simplified	Simplification criterion	Advantages	Limitations	Application domain
Feature simplification from triangular meshes [Date06]	Polygonal Mesh	Regular shaped blind holes, through holes and bosses	Quadric error based metric	Suppressed features can be recovered based on the LOD tree	Nested features not addressed	FEM model preparation
Feature removal from polyhedral model [Ribe01]	Polygonal Mesh	Cavity and protrusion types of features	Quadric error based metric	Uses general definition of feature	Implemented only for polyhedral models	FEM model preparation
Finite element based feature removal [Dabk94]	B-Rep	Axis-symmetric, plane, solid	Interactive	FE based features used	FE features like beam, pipe, shell, etc. not defined	FEM model preparation
Automatic fillet/round simplification [Zhu02]	B-Rep	Constant radii fillets and rounds	Interactive	Handles both ring and disc type topology; reinstatement of features possible	Variable radii fillets not covered; computationally intensive feature recognition step	Feature recognition
Blend removal from B-Rep [Venk02b]	B-Rep	Blends	Automatic	Complex blend networks with interacting features are handled	Variable radii blends not covered	Feature recognition
Detail Reduction for mesh generation [Taut01]	B-Rep	Blends	Hydraulic diameter of surface and volume	Size based metric; Portable across modeling systems	Roll-on and face edge blend not covered; bridge removal not covered	FE model preparation
Feature simplification using *face delete* operation [Venk02a]	B-Rep	Arbitrary shaped protrusion and depression	Interactive	Simplifies both protrusion and depression types of features	Could not handle cases where extension and contraction of adjacent faces can't patch the gap produced; under-constrained problems with multiple solutions not considered	Feature recognition
Feature simplification for freeform	B-Rep	Arbitrary shaped holes,	Interactive	Handles freeform	Suppressed features cannot be	FEM model preparation

surface models [Josh03]		fillets (constant and variable radii), free form surface features		surfaces; arbitrary shaped holes and variable radius fillets and rounds	recovered; cannot simplify features such as notches, lances, etc.	
Feature simplification using wrap-around, thinning and smoothing operators [Kim05, Seo05, Koo02]	B-Rep	Blends, chamfers, passages and concave regions	Based on summation of area of faces contained by features	Applicable to assembly models; interoperability and portability to commercial system	Rule based feature recognition results in overhead; mating information not preserved while simplifying assembly models	FEM model preparation, network model transmission and multi- resolution viewing

Table 4: Summary of techniques based on dimension reduction operators

Method	Input format	Features simplified	Simplification criterion	Advantages	Limitations	Application domain
Medial axis transform [Dona00]	B-Rep	Shell based features	Automatic	High aspect ratio regions can be identified; simplified models computationally efficient and comparable with original model w.r.t. FEA results	Only applicable for shells	FEM model preparation
Homotopy preserving medial axis simplification [Sud05]	Polygonal Mesh	Freeform features	Automatic	Results into homotopically equivalent medial axis	Pruning of unstable parts of medial axis may not be optimal	Mesh generation and shape analysis
θ-MAT based technique [Fosk03]	Polygonal Mesh	Protrusions and depressions	Based on separation angle	Computationally stable	Homotopy is not preserved	Mesh generation; shape analysis
Mid-surface abstraction [Reza96]	B-Rep	Prismatic features	Automatic	Material distribution effectively represented; volume preserved even in reduced model	Complex geometries like freeform bumps and depressions not handled	FEM, fluid flow simulation and feature recognition
Feature decomposition and selective mid-surface abstraction [Chon04]	B-Rep	Freeform features	Interactive	Model decomposition also performed for efficient mixed dimension modeling	Mid-surface extension and stitching operations may lead to errors in generated mid-surface	FEM model preparation

Table 5: Model simplification technique selection criteria

Input format	Application Domain	Features present in the model	Level of automation/flexibility desired	Suitable simplification techniques
B-Rep	FEA model simplification, fluid flow problems	Prismatic	Automatic	Mid-surface [Reza96], Wraparound/thinning/smoothing operators [Kim05], Detail Reduction for mesh generation [Taut01]
B-Rep	FEA model simplification, fluid flow problems	Prismatic	Interactive	Cell transformation based [Lee05a], Finite element based feature removal [Dabk94]
B-Rep	Feature recognition preprocessing	Freeform features	Interactive	Face delete operators [Venk02a], Blend removal [Venk02b], Fillet/Round simplification [Zhu02]
B-Rep	FEA model simplification	Shell based features	Automatic	Medial-axis transform [Dona00], [Fosk03]
B-Rep	FEA model simplification	Free-form features	Interactive	Selective mid-surface abstraction [Chon04], Feature simplification [Josh03], MCT based [Foca08], Homotopy Preserving medial axis technique [Sud05]
Mesh	FEA model simplification	Cavity and protrusion type	Automatic	Feature removal [Ribe01], Feature simplification [Date06] Surface entity based operators [Shef97], [Shef01], [Date05], [Inou01], [Vero98], Local modification in a mesh [Dey97], [Shep98]
Mesh	Collision detection	Protuberances and cavities	Automatic	Voxel based object simplification [He95], TDPS [Andu02]
Mesh	FEM model simplification	Freeform features	Automatic	Effective volume [Lee05b, Lee05c, Lee06a, Lee06b]
2D Image	FEM model simplification	Boundary edges and island type features	Automatic	Fourier transform based [Lee98]
Native feature	FEM model simplification	Freeform	Automatic	Cellular topology based PSM [Lee04]